

Индивидуальные задания для студентов первого курса по теме "Производные" и "Графики функций"¹

¹Красным цветом выделены *необязательные* задания.

Раздел 1.

ПРОИЗВОДНЫЕ

Задание № 1. Найти $f'(a)$ непосредственно по определению.

- | | |
|---|--|
| 1.1. $f(x) = \sin 5x, a = 0$ | 1.19. $f(x) = \frac{1}{(x-1)^2}, a = 2$ |
| 1.2. $f(x) = \cos 9x, a = 0$ | 1.20. $f(x) = \frac{1}{(2x+3)^2}, a = 0$ |
| 1.3. $f(x) = \operatorname{tg} x, a = \frac{\pi}{4}$ | 1.21. $f(x) = e^{-4x}, a = 0$ |
| 1.4. $f(x) = \operatorname{ctg} x, a = \frac{\pi}{4}$ | 1.22. $f(x) = \frac{1}{2 + \sin x}, a = 0$ |
| 1.5. $f(x) = 3x^2, a = 0$ | 1.23. $f(x) = \frac{1}{\sin(x + \frac{2\pi}{3})}, a = 0$ |
| 1.6. $f(x) = (x-2)^3, a = 0$ | 1.24. $f(x) = \frac{1}{\ln x}, a = e$ |
| 1.7. $f(x) = x^3, a = 0$ | 1.25. $f(x) = \frac{1}{x^2 + 2x + 3}, a = 0$ |
| 1.8. $f(x) = e^{2x}, a = 0$ | 1.26. $f(x) = x \sin x, a = 0$ |
| 1.9. $f(x) = \ln 2x, a = 1$ | 1.27. $f(x) = \frac{1}{2x - \sin x}, a = \frac{\pi}{2}$ |
| 1.10. $f(x) = 5^x, a = 0$ | 1.28. $f(x) = (x-1)^4, a = 0$ |
| 1.11. $f(x) = 3x - x^2, a = 0$ | 1.29. $f(x) = \frac{1}{(2x-1)^4}, a = 0$ |
| 1.12. $f(x) = 2 \sin x - x^2, a = 0$ | 1.30. $f(x) = \sin^3 x, a = 0$ |
| 1.13. $f(x) = 2 \operatorname{tg}(x-3), a = 3$ | 1.31. $f(x) = x^2 + 2x - 3, a = 1$ |
| 1.14. $f(x) = \sin 2(x+1), a = -1$ | 1.32. $f(x) = -\cos(x+1), a = -1$ |
| 1.15. $f(x) = (x-4)^2, a = 2$ | 1.33. $f(x) = (2x-3)^3, a = 1$ |
| 1.16. $f(x) = 5^{-x}, a = 1$ | 1.34. $f(x) = e^{x+2}, a = -2$ |
| 1.17. $f(x) = \frac{1}{\sin x}, a = \frac{\pi}{2}$ | 1.35. $f(x) = \operatorname{lg}(x-2), a = 4$ |
| 1.18. $f(x) = \frac{1}{\cos x}, a = 0$ | 1.36. $f(x) = \frac{2}{x(x+1)}, a = 1$ |

Задание № 2. Написать уравнение нормали (в вариантах 1 – 18) или касательной (в вариантах 19 – 36) к данной кривой в точке с абсциссой x_0 .

2.1. $y = \frac{6x - x^2}{3}, x_0 = 2$

2.19. $y = 2\sqrt{x} + 3\sqrt[4]{x}, x_0 = 16$

2.2. $y = 3x^2 - 2x + 1, x_0 = -1$

2.20. $y = 2\sqrt{x^3} - 16x, x_0 = 4$

2.3. $y = 4\sqrt[3]{x+1} + 3, x_0 = 7$

2.21. $y = \frac{x^{17} + 2}{x^5 + 1}, x_0 = 1$

2.4. $y = 2\sqrt[3]{x} - 4\sqrt{x}, x_0 = 64$

2.22. $y = \frac{3x}{x^2 + 3}, x_0 = 2$

2.5. $y = \sqrt[4]{x^3} - 35, x_0 = 16$

2.23. $y = 2x^2 + \frac{1}{2x^2}, x_0 = 1$

2.6. $y = 3(x+1) + \frac{1}{x+1}, x_0 = -3$

2.24. $y = \frac{2x^3 + 1}{5x^3 - 1}, x_0 = -2$

2.7. $y = \frac{2(x^2 + 1)}{3(x^4 + 2)}, x_0 = 1$

2.25. $y = -4x^4 + 3x^3 - 2x + 1, x_0 = -2$

2.8. $y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}, x_0 = 4$

2.26. $y = \frac{5(x^6 + 1)}{2x^2 + 1}, x_0 = 1$

2.9. $y = 2x + 3\sqrt{x}, x_0 = 16$

2.27. $y = \frac{1 + \sqrt[3]{x}}{2 - \sqrt[3]{x}}, x_0 = 1$

2.10. $y = 2x^3 + x^2 - x + 1, x_0 = 4$

2.28. $y = 5x^2 - 0.1x + 7, x_0 = -1$

2.11. $y = 2\sqrt{x} + 3x^2, x_0 = 1$

2.29. $y = 5\sqrt[5]{x} - 21, x_0 = 32$

2.12. $y = \frac{2 + x^2}{3 + x^3}, x_0 = 1$

2.30. $y = 2x^3 - 4\sqrt{x} - 11, x_0 = 4$

2.13. $y = 3\sqrt{x+2} + \frac{3}{\sqrt{x+2}}, x_0 = 7$

2.31. $y = \sqrt[3]{x^2 + 7} - x, x_0 = 1$

2.14. $y = \frac{8 - x^3}{8 + x^3}, x_0 = 1$

2.32. $y = \frac{x^7 - 2}{2x^2 + 1}, x_0 = -1$

2.15. $y = 5x^2 - 2x + 3, x_0 = -1$

2.33. $y = 5\sqrt{x-2} + x^2, x_0 = 6$

2.16. $y = \frac{x^{15} + 9}{3 - 2x^4}, x_0 = 1$

2.34. $y = \frac{2x + 1}{x^6 + x^2}, x_0 = -1$

2.17. $y = x^3 - 4x^2 + 1, x_0 = -1$

2.35. $y = -x^4 + 2x^3 - 8, x_0 = 2$

2.18. $y = 0.2x^5 - 3x, x_0 = -2$

2.36. $y = 2\sqrt[4]{x^3} + x^3, x_0 = 1$

Задание № 3. Составить уравнение касательной (в вариантах 1 – 18) или нормали (в вариантах 19 – 36) к данной кривой в точке, соответствующей значению параметра $t = t_0$.

$$3.1. \begin{cases} x(t) = a \cos t, \\ y(t) = a \sin t, \quad t_0 = \frac{\pi}{3} \end{cases}$$

$$3.2. \begin{cases} x(t) = \sqrt{2} \cos t, \\ y(t) = 2 \sin t, \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$3.3. \begin{cases} x(t) = t^2 - 3t, \\ y(t) = t^4 - 4t^2, \quad t_0 = 2 \end{cases}$$

$$3.4. \begin{cases} x(t) = 0.2t^4 - 1, \\ y(t) = 2t^3 + 2, \quad t_0 = 2 \end{cases}$$

$$3.5. \begin{cases} x(t) = \ln \operatorname{tg} t, \\ y(t) = \operatorname{ctg} t, \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$3.6. \begin{cases} x(t) = \frac{2 + t^2}{3 - t}, \\ y(t) = \frac{t}{2}, \quad t_0 = 1 \end{cases}$$

$$3.7. \begin{cases} x(t) = \frac{t}{t - 1}, \\ y(t) = \frac{t}{t + 1}, \quad t_0 = 3 \end{cases}$$

$$3.8. \begin{cases} x(t) = \sin 3t, \\ y(t) = \cos t, \quad t_0 = \frac{\pi}{3} \end{cases}$$

$$3.9. \begin{cases} x(t) = 2t^2 - t^3, \\ y(t) = 0.1t + 5, \quad t_0 = 3 \end{cases}$$

$$3.10. \begin{cases} x(t) = 2t^2 + t - 1, \\ y(t) = t^3 - 1, \quad t_0 = 2 \end{cases}$$

$$3.11. \begin{cases} x(t) = \sin^2 2t, \\ y(t) = \cos^2 2t, \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$3.12. \begin{cases} x(t) = 3 \operatorname{ctg} t, \\ y(t) = 2 \sin t + \cos t, \quad t_0 = \frac{\pi}{3} \end{cases}$$

$$3.19. \begin{cases} x(t) = \frac{1}{t}, \\ y(t) = t^2 + 1, \quad t_0 = 2 \end{cases}$$

$$3.20. \begin{cases} x(t) = \frac{1}{t^2 + 1}, \\ y(t) = 2t, \quad t_0 = 1 \end{cases}$$

$$3.21. \begin{cases} x(t) = \sin t + \cos t, \\ y(t) = 2 \operatorname{tg} t, \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$3.22. \begin{cases} x(t) = 3t \cos t, \\ y(t) = 3t \sin t, \quad t_0 = \frac{\pi}{2} \end{cases}$$

$$3.23. \begin{cases} x(t) = \sin^3 t - 1, \\ y(t) = \cos^3 t - 1, \quad t_0 = \frac{\pi}{6} \end{cases}$$

$$3.24. \begin{cases} x(t) = \ln \operatorname{tg} t, \\ y(t) = \cos^2 t, \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$3.25. \begin{cases} x(t) = \frac{1}{t^2}, \\ y(t) = t^2, \quad t_0 = 2 \end{cases}$$

$$3.26. \begin{cases} x(t) = \frac{1}{t^3 + 1}, \\ y(t) = t^3 + 2, \quad t_0 = 1 \end{cases}$$

$$3.27. \begin{cases} x(t) = 4 \cos t, \\ y(t) = 3 \sin t, \quad t_0 = \frac{4\pi}{3} \end{cases}$$

$$3.28. \begin{cases} x(t) = t - t^2, \\ y(t) = t^2 - 2t^3, \quad t_0 = 3 \end{cases}$$

$$3.29. \begin{cases} x(t) = \frac{2}{t} + 4t, \\ y(t) = \frac{2}{t + 1}, \quad t_0 = 1 \end{cases}$$

$$3.30. \begin{cases} x(t) = 3t^2 + 1, \\ y(t) = \frac{2}{t^2 + 1}, \quad t_0 = 2 \end{cases}$$

$$3.13. \begin{cases} x(t) = t - t^3, \\ y(t) = t^2 + 2t^4, \quad t_0 = 1 \end{cases}$$

$$3.14. \begin{cases} x(t) = t^3 - 1, \\ y(t) = t^2, \quad t_0 = 2 \end{cases}$$

$$3.15. \begin{cases} x(t) = \ln 2t, \\ y(t) = 2 \ln(4t - 1), \quad t_0 = \frac{1}{2} \end{cases}$$

$$3.16. \begin{cases} x(t) = e^t + 1, \\ y(t) = \cos t + 2, \quad t_0 = 0 \end{cases}$$

$$3.17. \begin{cases} x(t) = 3 - 4 \cos t, \\ y(t) = 1 + 2 \operatorname{tg} t, \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$3.18. \begin{cases} x(t) = a \cos t, \\ y(t) = b \sin t, \quad t_0 = \frac{\pi}{3} \end{cases}$$

$$3.31. \begin{cases} x(t) = 3t^4 - 2t^2, \\ y(t) = 2t + 5, \quad t_0 = 1 \end{cases}$$

$$3.32. \begin{cases} x(t) = \sin 2t + t, \\ y(t) = 2 \cos t + 2t, \quad t_0 = 0 \end{cases}$$

$$3.33. \begin{cases} x(t) = \frac{t+1}{t^1+1}, \\ y(t) = t^3 - 2, \quad t_0 = 1 \end{cases}$$

$$3.34. \begin{cases} x(t) = 2 + 2 \sin t, \\ y(t) = 3 - \cos 2t, \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$3.35. \begin{cases} x(t) = t^3 + 2t^2 - 1, \\ y(t) = \frac{1}{t^2 + 2}, \quad t_0 = 3 \end{cases}$$

$$3.36. \begin{cases} x(t) = 3t^2 - t^3 + 1, \\ y(t) = t^4 - 1, \quad t_0 = 2 \end{cases}$$

Задание № 4. Найти дифференциал функции.

$$4.1. y = \ln \frac{1}{x^2 + 1} + \operatorname{arctg} \sqrt{x}$$

$$4.19. y = \operatorname{arctg}(2 + \operatorname{tg} 2x)$$

$$4.2. y = x \arcsin x + \sqrt{x^3 + 1}$$

$$4.20. y = \sqrt[3]{x} - (1 + \sqrt[3]{x}) \arcsin \frac{1}{1 + \sqrt[3]{x^2}}$$

$$4.3. y = e^{2x}(\cos 3x + 2 \sin 3x)$$

$$4.21. y = \operatorname{arctg}(\ln(3 + x^2))$$

$$4.4. y = \arcsin e^{-x} + \ln \cos x$$

$$4.22. y = 3x^2 + \arccos \frac{x^2}{x^2 + 3}$$

$$4.5. y = x^2 \ln \sqrt{5 + x^2} - \sqrt{5 + x^2}$$

$$4.23. y = \lg \frac{\sqrt[3]{x^2 + 1}}{\sqrt[4]{x^2 + 2}}$$

$$4.6. y = \sin x \cdot \ln \operatorname{ctg} x - 2 \ln \operatorname{ctg} \frac{x}{2}$$

$$4.24. y = x \operatorname{arctg} \frac{x}{2} + \ln \cos x$$

$$4.7. y = \sqrt[5]{\frac{x-1}{x+5}}$$

$$4.25. y = \sqrt{x^2 + 11} - \operatorname{arctg} \frac{1}{x^2 + 11}$$

$$4.8. y = \frac{\cos \ln x + 2 \sin \ln x}{x^2 + 1}$$

$$4.26. y = \cos(e^{x^2} + 1) - \ln(e^{x^2} + 1)$$

$$4.9. y = \operatorname{arctg} \frac{x^3 + 1}{x}$$

$$4.27. y = \cos(2 \ln(x^4 + 1)) - \sin(2 \ln(x^4 + 1))$$

$$4.10. y = \arccos \frac{x}{2} + x \sqrt{16 - x^2}$$

$$4.28. y = \frac{\sqrt[3]{1-x}}{\sqrt[4]{\cos^2(x+1)}}$$

$$4.11. y = \lg \sqrt{\sin x} + x \operatorname{tg} x$$

$$4.29. y = \frac{\sin x}{x} - \ln \frac{x^2 + 1}{x^2 + 7}$$

$$4.12. y = \log_2 \frac{\sqrt{x^2 + 1} + x}{2x}$$

$$4.30. y = (\sqrt{x^2 - 1} - 2)e^{2\sqrt{x^2 - 1}}$$

$$4.13. y = \operatorname{arctg} \ln x + e^{\operatorname{arctg} x}$$

$$4.31. y = \log_2 \sqrt[3]{x^2 + 8} - 2 \arcsin \frac{x - \ln x}{\sin x}$$

$$4.14. y = \sqrt[3]{1 + x^2} + \arcsin \frac{x}{x + 1}$$

$$4.32. y = \operatorname{arctg}^2 \frac{\ln x}{\sqrt{x^2 + 2}}$$

$$4.15. y = \ln \sqrt{2 + 3x^2} - (x + 1) \arcsin x$$

$$4.33. y = \frac{\sin^3(x^2 - 3)}{\sqrt{x^3 - 2}} + x \lg(x^3 + 4)$$

$$4.16. y = \sqrt[3]{\frac{x^2 + 1}{2x^2 - 3}}$$

$$4.34. y = \ln \arcsin(x^3 - 1) + x \sin \sqrt{x}$$

$$4.17. y = \ln(e^{2x} + \sqrt[3]{e^{3x} - 1}) + \operatorname{arctg} e^{3x}$$

$$4.35. y = \frac{x^4 - 1}{\sqrt{2x + 2}} - \sin(e^x + x^2)$$

$$4.18. y = \sin \frac{x^2}{x^2 + 4} + \arcsin \frac{1}{x^2 + 4}$$

$$4.36. y = \operatorname{arctg} \sin \sqrt[5]{\frac{x+1}{x-2}} - 2 \lg \sqrt{x^3}$$

Задание № 5. Вычислить приближенно с помощью дифференциала.

5.1. $y = \sqrt[5]{x}$, $x = 31.98$

5.2. $y = \arcsin x$, $x = 0.04$

5.3. $y = \operatorname{arctg} x$, $x = 1.02$

5.4. $y = \sqrt{2x+3}$, $x = 2.97$

5.5. $y = \frac{1}{\sqrt{2x}}$, $x = 7.99$

5.6. $y = x^5$, $x = 2.02$

5.7. $y = \sqrt{1+\sin x}$, $x = 0.02$

5.8. $y = \sqrt{4+\operatorname{tg} x}$, $x = 0.001$

5.9. $y = \sin x$, $x = 0.251\pi$

5.10. $y = \cos x$, $x = \frac{1003}{3000}\pi$

5.11. $y = \sqrt[3]{2x-2}$, $x = 5.001$

5.12. $y = \sqrt{3x+4}$, $x = 3.999$

5.13. $y = \operatorname{tg} x$, $x = 0.253\pi$

5.14. $y = \frac{1}{\sqrt{1+2x}}$, $x = 0.01$

5.15. $y = \sqrt[5]{x^3}$, $x = 32.003$

5.16. $y = \sqrt{2x^2+3x+2}$, $x = 1.97$

5.17. $y = \sqrt[3]{2x+\cos x+7}$, $x = 0.01$

5.18. $y = \sqrt{8x+\operatorname{tg} \frac{\pi x}{4}}$, $x = 1.01$

5.19. $y = \sqrt{2x^2+1}$, $x = 2.001$

5.20. $y = \frac{1}{\sqrt{4+\operatorname{tg} x}}$, $x = 0.01$

5.21. $y = \sqrt[3]{x}$, $x = 7.82$

5.22. $y = \frac{1}{\sqrt[4]{x^3}}$, $x = 1.04$

5.23. $y = x^4$, $x = 4.01$

5.24. $y = \ln(7x+1)$, $x = 0.03$

5.25. $y = e^x$, $x = 0.01$

5.26. $y = \frac{1}{\sqrt[4]{3x+4}}$, $x = 4.001$

5.27. $y = \sqrt{2x}$, $x = 8.002$

5.28. $y = (x+1)^4$, $x = 3.01$

5.29. $y = \sqrt{1+\sin x+\operatorname{tg} x}$, $x = 0.001$

5.30. $y = \sqrt[3]{2x^2+5x+9}$, $x = 2.001$

5.31. $y = 2 \operatorname{tg} \pi(x-2)$, $x = 2.01$

5.32. $y = \frac{1}{\sqrt{x-1}}$, $x = 4.97$

5.33. $y = \sqrt{x^2+3x}$, $x = 0.99$

5.34. $y = \ln(x^2-3)$, $x = 1.98$

5.35. $y = (x+1)^3$, $x = 1.005$

5.36. $y = \sqrt[3]{x^2+2}$, $x = 5.02$

Задание № 6. Найдите y' .

$$6.1. y = \frac{2(3x^3 + 4x^2 - x - 2)}{15\sqrt{1+x}}$$

$$6.2. y = \frac{(2x^2 - 1)\sqrt{1+x^2}}{3x^3}$$

$$6.3. y = \frac{x^4 - 8x^2}{2(x^2 - 4)}$$

$$6.4. y = \frac{2x^2 - x - 1}{3\sqrt{2+4x}}$$

$$6.5. y = \frac{(1+x^8)\sqrt{1+x^2}}{12x^{12}}$$

$$6.6. y = \frac{x^2}{2\sqrt{1-3x^4}}$$

$$6.7. y = \frac{(x^2 - 6)\sqrt{(4+x^2)^3}}{120x^5}$$

$$6.8. y = \frac{(x^2 - 8)\sqrt{x^2 - 8}}{6x^3}$$

$$6.9. y = \frac{4 + 3x^3}{x\sqrt[3]{(2+x^3)^2}}$$

$$6.10. y = \sqrt[3]{\frac{(1+x^2)^2}{x^{\frac{3}{2}}}}$$

$$6.11. y = \frac{x^6 + x^3 - 2}{\sqrt{1-x^3}}$$

$$6.12. y = \frac{(x-2)\sqrt{4+x^2}}{24x^2}$$

$$6.13. y = \frac{1+x^2}{2\sqrt{1+2x^2}}$$

$$6.14. y = \frac{\sqrt{x-1}(3x+2)}{4x^2}$$

$$6.15. y = \frac{\sqrt{(1+x^2)^3}}{3x^3}$$

$$6.16. y = \frac{x^6 + 8x^3 - 128}{\sqrt{8-x^3}}$$

$$6.17. y = \frac{\sqrt{2x+3}(x-2)}{x^2}$$

$$6.18. y = (1-x^2)\sqrt[5]{x^3 + \frac{1}{x}}$$

$$6.19. y = \frac{(2x^3 + 3)\sqrt{x^2 - 3}}{9x^3}$$

$$6.20. y = \frac{x-1}{(x^2+5)\sqrt{x^2+5}}$$

$$6.21. y = \frac{(2x+1)\sqrt{x^2-x}}{x^2}$$

$$6.22. y = 2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$$

$$6.23. y = \frac{1}{(x+2)\sqrt{x^2+4x+5}}$$

$$6.24. y = 3\frac{\sqrt[3]{x^2+x+1}}{x+1}$$

$$6.25. y = \frac{3\sqrt[3]{(x+1)(x-1)^2}}{x+2}$$

$$6.26. y = \frac{(x+7)\sqrt{x^2+2x+7}}{2x}$$

$$6.27. y = \frac{x\sqrt{x+1}}{x^2+3x+5}$$

$$6.28. y = \frac{x^2+2}{2\sqrt{1-x^4}}$$

$$6.29. y = \frac{(x+3)\sqrt{2x-1}}{2x+7}$$

$$6.30. y = \frac{3x+\sqrt{x}}{\sqrt{x^2+2}}$$

$$6.31. y = \frac{x^2 + \sqrt[3]{2x}}{1-2x^2-x^3}$$

$$6.32. y = \frac{1 + \sqrt[4]{x^2-5}}{\sqrt{x-3}}$$

$$6.33. y = \frac{2+x}{(1-x)\sqrt{3x^2+x}}$$

$$6.34. y = \frac{3}{(x^3-5)\sqrt[5]{x+2}}$$

$$6.35. y = \frac{x-4}{\sqrt[3]{(x+2)(4-5x)}}$$

$$6.36. y = \frac{(x^2-3)\sqrt{1-3x^3}}{2x-x^2}$$

Задание № 7. Найти y' .

$$7.1. y = \frac{1}{4}2^x(x^3 + x^2 + 2)$$

$$7.2. y = \sqrt{2 + 3e^{2x}} + \arccos e^{2x}$$

$$7.3. y = 2e^{\sqrt{x}}(\sqrt[5]{x^3} - 5\sqrt[3]{x^5} + 10x - 30\sqrt[3]{x^2} + 60\sqrt[3]{x} + 30)$$

$$7.4. y = \ln(\sqrt{1 - e^x} - 1) \ln \sqrt{1 - e^x - x} + (x - 1)\sqrt{1 - e^x}$$

$$7.5. y = \operatorname{arctg}(e^{-x} - e^x)$$

$$7.6. y = ((x^2 - 1) \cos x + (x - 1)^3 \sin 2x)e^x$$

$$7.7. y = e^{\cos x} \left(x^2 - \frac{1}{\sin x} \right)$$

$$7.8. y = \ln \frac{e^x + 1 + \sqrt{1 - e^{2x} - e^{3x}}}{e^x - 1 + \sqrt{1 - e^{2x} - e^{3x}}}$$

$$7.9. y = 5e^{\sqrt[5]{x}}(\sqrt[4]{x^3} - 2\sqrt[4]{x} + 1)$$

$$7.10. y = \frac{1}{n\sqrt{ab}} \operatorname{arctg} \left(e^{nx} \sqrt[3]{\frac{a}{b}} \right)$$

$$7.11. y = \frac{e^{x^2}}{x^2 + 1}$$

$$7.12. y = \frac{1}{e^{\frac{x}{3}} + 1} + x$$

$$7.13. y = x^2 - \log_2(2^x + 1) - 2^{-\frac{x}{2}} \operatorname{arctg} 2^{\frac{x}{2}} - \operatorname{arctg}^2 2^{\frac{x}{2}}$$

$$7.14. y = \ln(\arcsin e^{-x} + e^x) + \sqrt{1 + e^x}$$

$$7.15. y = 3 \operatorname{arctg} e^{\frac{x}{3}} - 3 \ln(\sqrt{e^{\frac{x}{3}} + 1}(1 + e^{\frac{x}{3}})) + 3x$$

$$7.16. y = x + \frac{1}{3^x + 1} + 2 \ln(3^x + 3)$$

$$7.17. y = 1 + \ln(1 + e^x + \sqrt{1 + e^x})$$

$$7.18. y = e^{2x}(2 \sin 3x - 3 \cos 3x)(e^3 - e^2)$$

$$7.19. y = \frac{1}{3}e^{3x}(3 - \sin 3x)$$

$$7.20. y = e^{3x}(3 \sin 2x - 2 \cos 2x)(e^3 - e^2)$$

$$7.21. y = \frac{1}{2} \operatorname{arctg} \frac{e^{2x} - 2}{3}$$

$$7.22. y = \frac{35h^2x}{2^{3x}} - \frac{1}{5} \log_2(e^{3x} - 5x)$$

$$7.23. y = \frac{1}{\ln 3} 2(\sqrt{3^x - 1} - \arcsin \sqrt{3^x - 1})$$

$$7.24. y = \frac{1}{\ln 3} \ln \frac{1 + 3^x}{3^x - 1}$$

$$7.25. y = 4 \operatorname{arctg} e^{4x} + \frac{1}{4} \ln(e^{4x} + 1)$$

$$7.26. y = \frac{3}{2} \sqrt{\arcsin^5 e^{3x}}$$

$$7.27. y = \frac{8e^{2x} + 7e^x + 6}{3(e^x + 1)^2}$$

$$7.28. y = \sqrt{\arccos^3 e^{4x}}$$

$$7.29. y = \frac{a \cos 26x + 26 \sin 26x}{2(a^2 + 3b^2)}$$

$$7.30. y = \sqrt{e^{2x} + 1} + \log_2 \frac{\sqrt{e^{2x} + 1} - 2}{\sqrt{e^{2x} + 1} + 2}$$

$$7.31. y = \ln^3 \left(e^{3x} + \frac{x^2}{x - 3} \right)$$

$$7.32. y = \sin^3 \left(\ln \frac{x}{5 + x} + e^{2x-3} \right) + \frac{x^2}{3^x}$$

$$7.33. y = \sqrt[5]{\cos^3(2^x + \log_3(4x - 5))}$$

$$7.34. y = \arcsin \frac{\ln(2x - 3)}{\log_2(2 - 5x)} - e^{\frac{3x}{x-1}}$$

$$7.35. y = \frac{4^{2-3x} + 3^{4-3x}}{\arccos(x^2 - 3^2)}$$

$$7.36. y = \operatorname{arctg}^2(e^{x^3} + \log_4(x^4 - x^2 + 1))$$

Задание № 8. Найти y' .

$$8.1. y = \lg \frac{\sin \frac{1}{x}}{\lg x}$$

$$8.2. y = \ln \ln \cos\left(1 - \frac{1}{x^2}\right)$$

$$8.3. y = \log_2\left(\arcsin \frac{1}{\sqrt[3]{x}}\right)$$

$$8.4. y = \lg \frac{x\sqrt{10} - \sqrt{1+x^3}}{x\sqrt{10} + \sqrt{1+x^3}}$$

$$8.5. y = \operatorname{arctg}(\ln \sqrt{1+e^x})$$

$$8.6. y = \log_3 \frac{1}{\sqrt[3]{1+x^3}}$$

$$8.7. y = \ln \sin \frac{x+1}{x-1}$$

$$8.8. y = \log_3 \log_2 \sin x$$

$$8.9. y = \lg \operatorname{tg} \frac{x-2}{x+2}$$

$$8.10. y = a^{e^{\sqrt{3}}} + \frac{1}{\sqrt{3}} \lg \frac{\sqrt{3-x}}{x+\sqrt{3}}$$

$$8.11. y = \ln^2 \sin\left(\frac{\pi}{2} + \frac{x}{3}\right)$$

$$8.12. y = \log_2 \frac{x^2}{1+x^2}$$

$$8.13. y = \lg \frac{x^2+a^2}{x^2-a^2}$$

$$8.14. y = \log_3 \frac{x^3}{\sqrt{ax^4-1}}$$

$$8.15. y = \lg(ax + \sqrt{a^2x^2+a})$$

$$8.16. y = \sqrt[3]{x} - 2 \ln(3 - \sqrt[3]{x})$$

$$8.17. y = \ln^3 \ln^2 \ln x$$

$$8.18. y = \lg \frac{\sin \frac{x}{2} + \sqrt{3}}{\cos \frac{x}{2} - \sqrt{3}}$$

$$8.19. y = \ln(e^{2x} - \sqrt[3]{e^x+1})$$

$$8.20. y = \ln(\sqrt{ax^2+b^2} + ax^3)$$

$$8.21. y = \operatorname{arctg} \sqrt{1 + \ln(1+e^x)}$$

$$8.22. y = \frac{\ln(\sqrt{1+\operatorname{tg} x} + \sqrt{3}\operatorname{tg} x)}{\sqrt{3}}$$

$$8.23. y = \frac{x}{3}(\sin \ln x - \cos \ln x)$$

$$8.24. y = \log_{11} \log_6 \operatorname{ctg} x^5$$

$$8.25. y = \log_2 \log_3 \operatorname{tg} x^2$$

$$8.26. y = \ln \sqrt[4]{\frac{1+4x}{\sin^3 x}}$$

$$8.27. y = \ln^5(\sin 5x + 1)$$

$$8.28. y = \lg^3(x^3 + \sin 3x)$$

$$8.29. y = \ln(\sqrt{x^2+1} + \sqrt[3]{x^2+1})$$

$$8.30. y = \sqrt{x+a} - \ln(\sqrt{x+a} - \sqrt{a})$$

$$8.31. y = -\log_2^3 \cos\left(\pi - \frac{x}{2}\right)$$

$$8.32. y = \lg^2 \frac{x^2 + \sin x}{x}$$

$$8.33. y = \arcsin \ln \frac{x+1}{x-1}$$

$$8.34. y = \log_3 \sqrt[3]{\frac{1+3x}{2-x}}$$

$$8.35. y = \ln^3 \sin \log_2 \frac{x+1}{2}$$

$$8.36. y = \cos^2 \ln \sqrt[3]{x-8}$$

Задание № 9. Найти y' .

$$9.1. y = \operatorname{arctg}\left(\frac{x\sqrt{3}}{x+2} + 1\right) - \sqrt{5}\ln(x-1)$$

$$9.2. y = 2\sqrt{x} - (x - 3\sqrt{x}) \arcsin \frac{\sqrt{x}}{2 + \sqrt{x}}$$

$$9.3. y = \frac{x}{\sqrt{2}} \arccos \frac{x^2 + 1}{x\sqrt{2}}$$

$$9.4. y = \frac{1}{5} \ln(4x^2 + 1) + \operatorname{arctg} \frac{x}{3\sqrt{4x^2 + 1}}$$

$$9.5. y = \operatorname{arctg} \frac{1 + \sqrt{x}}{1 - \sqrt{x}}$$

$$9.6. y = 2x - (2x^2 + 1) \operatorname{arctg} \frac{x}{\sqrt{2}}$$

$$9.7. y = \frac{8}{5} \arcsin \frac{\sqrt{x}}{2} + \frac{1}{3} \operatorname{arctg} \sqrt{x} + \sqrt{x}$$

$$9.8. y = x^2 \arccos \sqrt{x^3 + 1} + \sqrt{x^3 + 1}$$

$$9.9. y = \operatorname{arctg} \frac{x^2 + 1}{x^2 - 1} - \frac{3}{2} \ln \sqrt{x}$$

$$9.10. y = \operatorname{arctg} \frac{x + 1}{(x + 2)\sqrt{x}}$$

$$9.11. y = \frac{2x}{x-1} \sqrt{x^2 + 3x - 1} + \frac{1}{3} \operatorname{arctg} \sqrt{\frac{x-1}{2}}$$

$$9.12. y = \frac{(x+1) \operatorname{arctg} \sqrt{x}}{x^2}$$

$$9.13. y = \arcsin \sqrt{x-1} - \frac{x-2}{3} \sqrt{5x-x^2-1}$$

$$9.14. y = \arcsin \sqrt{x-1} - \frac{x-2}{3} \sqrt{5x-x^2-1}$$

$$9.15. y = \frac{\arccos x}{3x^3} - \frac{1}{4} \sqrt{\frac{1}{x} + 1}$$

$$9.16. y = \arcsin \frac{\sqrt{x}}{3}$$

$$9.17. y = \arcsin \frac{x^3}{3} + \frac{3}{x}$$

$$9.18. y = \operatorname{arctg} x - \arccos \sqrt{\frac{x+1}{x}}$$

$$9.19. y = \frac{1}{2\sqrt{x}} + \arccos \sqrt{x}$$

$$9.20. y = \frac{x+3}{2} \sqrt{x+1} - 2 \arccos x$$

$$9.21. y = \frac{(x+2) \operatorname{arctg} \sqrt{x+2}}{x^2}$$

$$9.22. y = \frac{x^2}{2} \arcsin x - x\sqrt{x+2}$$

$$9.23. y = \frac{x^2+1}{2} \sqrt{x^2-x+1} - 7 \arcsin \sqrt{\frac{x-7}{5}}$$

$$9.24. y = \frac{1}{3} \ln \frac{1-x}{1+x} - \frac{1}{3} \operatorname{arctg} x$$

$$9.25. y = \arccos \frac{x-2}{\sqrt{x}}$$

$$9.26. y = \sqrt{\frac{2}{3}} \operatorname{arctg} \frac{x-1}{\sqrt{3x}}$$

$$9.27. y = \arcsin \frac{\sqrt{1+x}-1}{x^2}$$

$$9.28. y = \frac{9}{8} \operatorname{arctg} \frac{3x+1}{3}$$

$$9.29. y = \operatorname{arctg} \frac{\sin x - \cos x}{\sqrt{3}}$$

$$9.30. y = \arccos \frac{\operatorname{tg} x - 1}{\sqrt{2}}$$

$$9.31. y = \arcsin \frac{x^3+1}{4-x^2} + \frac{2}{x}$$

$$9.32. y = \sqrt{x} \arccos \sqrt{x+1} - 2 \ln \sqrt{2x}$$

$$9.33. y = \frac{2x}{3} \operatorname{arctg} \sqrt[3]{\frac{x}{x+1}}$$

$$9.34. y = \frac{(1-x^2) \operatorname{arctg} \sqrt[3]{x}}{2x+3}$$

$$9.35. y = \arcsin \frac{1 + \sqrt[3]{2x}}{1 - \sqrt{x}}$$

$$9.36. y = \frac{1}{2\sqrt{x}} - \arccos \frac{x+3}{2-x}$$

Задание № 10. Найти y' .

10.1. $y = \frac{1}{2}(\ln \sin 3x)^x$

10.2. $y = 2^{x^2}$

10.3. $y = x^{e^x}$

10.4. $y = (x + 1)^{\operatorname{tg} x}$

10.5. $y = x^{e^{\cos x}}$

10.6. $y = (\sin x)^{\ln x}$

10.7. $y = x^{x^3}$

10.8. $y = 3^x \cdot x^{2x}$

10.9. $y = \log_x 2^x$

10.10. $y = \log_x(x^2 + 1)$

10.11. $y = (\operatorname{tg} \sqrt{x})^{e^x}$

10.12. $y = x^{e^{\operatorname{tg} x}}$

10.13. $y = 9^x \cdot x^{9x}$

10.14. $y = x^{2x} \cdot \ln x$

10.15. $y = (\sin x)^{x^2}$

10.16. $y = (x + 1)^{\sin x}$

10.17. $y = (1 - x)^{\sin x}$

10.18. $y = (x^2 + 1)^{\operatorname{tg} x}$

10.19. $y = \log_{x+1} x^2$

10.20. $y = \log_{x^2}(x + 1)$

10.21. $y = (x^3 + 1)^{x^3}$

10.22. $y = (1 + x^2)^{x^2}$

10.23. $y = (x \sin x)^x$

10.24. $y = (x \operatorname{tg} x)^{\frac{x}{2}}$

10.25. $y = (\operatorname{tg} x)^{e^x}$

10.26. $y = x^{\arcsin x}$

10.27. $y = x^{\operatorname{arctg} x}$

10.28. $y = (\ln x)^{\arccos x}$

10.29. $y = (\sin \sqrt[3]{x})^{\ln x}$

10.30. $y = (\operatorname{arctg} x)^{\arcsin x}$

10.31. $y = (\operatorname{arcctg} x)^{x^2+1}$

10.32. $y = (\sin x)^{x \ln x}$

10.33. $y = (x^2 + 1)^{\sin \sqrt{x}}$

10.34. $y = (2x + e^2)^{\cos x}$

10.35. $y = (\sin \log_2 x)^{\cos x}$

10.36. $y = (\operatorname{arctg} 3x)^{x^3}$

Задание № 11. Найти y' .

$$11.1. y = \ln \frac{1 + \sqrt{3x^2 + 1}}{x}$$

$$11.2. y = \sqrt{47x^2 + 1} \operatorname{arctg} 9x$$

$$11.3. y = \ln(e^{2x} + \sqrt{e^{2x} + 1})$$

$$11.4. y = \operatorname{arctg} \ln \sqrt{x}$$

$$11.5. y = (x + 1)^4 \arccos \frac{1}{2x + 1}$$

$$11.6. y = \ln(3x + \sqrt{9x^2 + 1})$$

$$11.7. y = \ln \frac{2 + \sqrt{-x^2 - 9x + 1}}{x + 2}$$

$$11.8. y = \operatorname{arctg}(5x) \cdot \sqrt{5x - 1}$$

$$11.9. y = \frac{x}{x + 1} + \log_2 \frac{2x + 1}{x}$$

$$11.10. y = \arcsin e^{-3x}$$

$$11.11. y = \ln(e^{2x} + \sqrt{e^x - 1})$$

$$11.12. y = \sqrt{x^2 - x} \arccos \sqrt{x - 1}$$

$$11.13. y = \ln \frac{\sin x + \cos x}{\operatorname{arctg} x}$$

$$11.14. y = \ln(5 + x + \sqrt{x + 5})$$

$$11.15. y = \sqrt{3x^2 + x - 1} \cdot \lg(3x^2 - 1)$$

$$11.16. y = \arccos \ln x^2$$

$$11.17. y = \frac{x - 2}{x + 3} \sin e^{x-2}$$

$$11.18. y = (x - 1) \sqrt[3]{x - 2} \cdot \ln^4 x$$

$$11.19. y = \sqrt{x^3 - 1} \operatorname{arctg} \sqrt[3]{x}$$

$$11.20. y = \ln \operatorname{arctg} \sqrt{x}$$

$$11.21. y = e^{-3x \arcsin e^{3x}}$$

$$11.22. y = 3x - 2 \ln(1 - 2\sqrt{x})$$

$$11.23. y = \frac{x^3}{27} \arccos \frac{7}{x} + \ln \frac{1}{x}$$

$$11.24. y = \ln \frac{1}{x^2} + \operatorname{arctg} \frac{3}{x}$$

$$11.25. y = \ln(1 + \sqrt{x} - \sqrt{x^2 - 9})$$

$$11.26. y = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{5x}{\sqrt{2}}$$

$$11.27. y = \frac{x^3}{9} \arccos \frac{9}{x^3}$$

$$11.28. y = \frac{x^2}{5} \operatorname{arctg} \frac{5}{x^2}$$

$$11.29. y = \log_2 e^{2x} + \ln 2x$$

$$11.30. y = e^{x^2} \cdot \arcsin x^2$$

$$11.31. y = \operatorname{arctg} \log_2 \sin x$$

$$11.32. y = \log_3 \arcsin \sqrt{1 - x}$$

$$11.33. y = \frac{x^2}{1 - x} + \log_4 \frac{x}{x^2 - 3}$$

$$11.34. y = \frac{x^2 + 1}{x} \sin \ln x^2$$

$$11.35. y = e^{1-x^2} \cdot \operatorname{arctg} \frac{x + 1}{\sqrt{x}}$$

$$11.36. y = \frac{e^{2x}}{x} \operatorname{arctg} \log_5 \frac{x}{3}$$

Задание № 12. Найти y' .

$$12.1. y = x \arcsin \sqrt{\frac{x+2}{2}} - \sqrt{x+2}$$

$$12.2. y = \sqrt{x^2 - 4} \cdot \arccos \frac{1}{x+1}$$

$$12.3. y = 10^x \cdot (10 - x^2)$$

$$12.4. y = x^2 \sin x^2 + x^2$$

$$12.5. y = \frac{1}{x^2} + \operatorname{tg} \frac{1}{x^2}$$

$$12.6. y = 2\sqrt{1-x^2} + e^x \sqrt{1-x^2}$$

$$12.7. y = x\sqrt{\arcsin x}$$

$$12.8. y = \sqrt{(x+3)(x-2)} + \frac{1}{e^{2x}}$$

$$12.9. y = x \ln(\sqrt{1-x} + \sqrt{1+x})$$

$$12.10. y = \frac{1}{2}(\arccos x - x^2)7^x$$

$$12.11. y = \arcsin \frac{3}{\sqrt{x-3}}$$

$$12.12. y = \ln \sqrt[5]{\frac{x-2}{x-1}} + 5^x$$

$$12.13. y = \left(\frac{1}{2} - \frac{1}{x^2-1} \right) \operatorname{arctg} x$$

$$12.14. y = -\frac{\ln x}{\sqrt{x^3+1}}$$

$$12.15. y = \ln(\sqrt{x+1} + 2)e^{\sqrt{x+1}}$$

$$12.16. y = \sqrt{4x^2 + 12x - 16} \operatorname{arctg} \frac{\sqrt{x}}{3}$$

$$12.17. y = \frac{1}{6} \ln \frac{x^3 - x + 1}{(x+1)^3}$$

$$12.18. y = \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{\sqrt{2}}{2x^2+1}$$

$$12.19. y = \frac{1}{2} \ln \frac{\sqrt{x^2-1} - x}{\sqrt{x^2+1} + 1}$$

$$12.20. y = 3 \ln(\sqrt{3+x} + \sqrt{x-3})$$

$$12.21. y = \arcsin \frac{2x+1}{x^2} + \frac{x^2}{\sqrt{17}}$$

$$12.22. y = x^3 \operatorname{arctg} x^3 + \sqrt[3]{x^3+1}$$

$$12.23. y = 3 \ln \frac{x}{1 - \sqrt{x^2+4}}$$

$$12.24. y = \frac{x \operatorname{arctg} x}{\sqrt{x^2+x}}$$

$$12.25. y = \sqrt{(x+1)(2-x)} + 2 \ln \sqrt{x+1}$$

$$12.26. y = \ln(x + \sqrt{1+x^2}) + \frac{\sqrt{1+x^2}}{3x}$$

$$12.27. y = 2^{x+1} \cdot \ln(x+1)$$

$$12.28. y = \operatorname{arctg}^2 x \cdot \sqrt{\arccos x}$$

$$12.29. y = \sqrt{x} \ln \sqrt[5]{x}$$

$$12.30. y = \arcsin \sqrt[3]{x} \cdot \sqrt[3]{\arcsin x}$$

$$12.31. y = e^{\sqrt{1-x}} \cdot \operatorname{arctg} \sqrt[3]{x^2+1}$$

$$12.32. y = \sqrt{x^2+1} \cdot \lg^3 \arcsin x$$

$$12.33. y = 2x \ln \frac{\sqrt{x}}{1 + \sqrt{x^2-3}}$$

$$12.34. y = \frac{x}{5} \log_{\frac{1}{2}} \operatorname{arctg} \sqrt[3]{x}$$

$$12.35. y = \arcsin \frac{2x}{\sqrt[3]{\ln x}}$$

$$12.36. y = \sqrt{x+1} - 5 \ln^2 \sqrt{x+2}$$

Задание № 13. Найти производную y'_x .

$$13.1. \begin{cases} x(t) = \ln \sqrt{1-t^2}, \\ y(t) = t \arcsin t \end{cases}$$

$$13.19. \begin{cases} x(t) = \ln \frac{1}{t}, \\ y(t) = \arcsin(1-t^2) \end{cases}$$

$$13.2. \begin{cases} x(t) = e^{\sin t}, \\ y(t) = t - \operatorname{tg} t \end{cases}$$

$$13.20. \begin{cases} x(t) = \sqrt{t^2+1}, \\ y(t) = \frac{1}{\sqrt{1-t^2}} \end{cases}$$

$$13.3. \begin{cases} x(t) = \sqrt{t-t^2}, \\ y(t) = \operatorname{arctg} \sqrt{\frac{1-t}{t}} \end{cases}$$

$$13.21. \begin{cases} x(t) = \operatorname{arctg} e^t, \\ y(t) = \sqrt{e^t+2} \end{cases}$$

$$13.4. \begin{cases} x(t) = \sqrt{t}, \\ y(t) = \sqrt{1-t} \arcsin \sqrt{t} \end{cases}$$

$$13.22. \begin{cases} x(t) = \ln \sin t, \\ y(t) = \operatorname{ctg} t \end{cases}$$

$$13.5. \begin{cases} x(t) = \frac{t^2 \ln t}{1-t^2}, \\ y(t) = \ln \sqrt{1-t^2} \end{cases}$$

$$13.23. \begin{cases} x(t) = \operatorname{tg}(2e^{\frac{t}{2}}), \\ y(t) = \ln \operatorname{tg} e^t \end{cases}$$

$$13.6. \begin{cases} x(t) = \operatorname{arctg}(t+1), \\ y(t) = \arcsin \sqrt{1-t^2} \end{cases}$$

$$13.24. \begin{cases} x(t) = \sqrt{2t+t^3}, \\ y(t) = \operatorname{arctg} t \end{cases}$$

$$13.7. \begin{cases} x(t) = \ln \sqrt{1-\sin t}, \\ y(t) = \ln \cos t \end{cases}$$

$$13.25. \begin{cases} x(t) = \lg(t + \sqrt{t^2+1}), \\ y(t) = t\sqrt{t^2+\sqrt{2}} \end{cases}$$

$$13.8. \begin{cases} x(t) = \ln \sin t, \\ y(t) = \frac{1}{\cos^2 t} \end{cases}$$

$$13.26. \begin{cases} x(t) = \operatorname{arctg}(\operatorname{tg} t), \\ y(t) = \arcsin(\cos t) \end{cases}$$

$$13.9. \begin{cases} x(t) = t\sqrt{t+1}, \\ y(t) = \ln(1+\sqrt{t+1}) \end{cases}$$

$$13.27. \begin{cases} x(t) = t^2+t+1, \\ y(t) = \sin(t^2+1) \end{cases}$$

$$13.10. \begin{cases} x(t) = \operatorname{arctg} t, \\ y(t) = \ln \sqrt{t} \end{cases}$$

$$13.28. \begin{cases} x(t) = \sin t + \cos t, \\ y(t) = \arcsin t + \arccos t \end{cases}$$

$$13.11. \begin{cases} x(t) = \arccos \sqrt{t+1}, \\ y(t) = \ln(t+1) \end{cases}$$

$$13.29. \begin{cases} x(t) = \sin \ln t, \\ y(t) = \ln \sin t \end{cases}$$

$$13.12. \begin{cases} x(t) = \arccos \sqrt{t}, \\ y(t) = \sqrt{1+\sqrt{t}} \end{cases}$$

$$13.30. \begin{cases} x(t) = t^3+1, \\ y(t) = \sin^3 t + 1 \end{cases}$$

$$13.13. \begin{cases} x(t) = \arcsin^2 t, \\ y(t) = t\sqrt{1-t} \end{cases}$$

$$13.31. \begin{cases} x(t) = \lg(t^2 + \sqrt{t+1}), \\ y(t) = \sin(1 + \sqrt{t}) \end{cases}$$

$$13.14. \begin{cases} x(t) = \frac{1}{\ln t}, \\ y(t) = \ln \frac{1}{t} \end{cases}$$

$$13.15. \begin{cases} x(t) = \arcsin \frac{1}{t}, \\ y(t) = \arccos \frac{1}{t} \end{cases}$$

$$13.16. \begin{cases} x(t) = 1 + \cos^2 t, \\ y(t) = \cos t \end{cases}$$

$$13.17. \begin{cases} x(t) = \arcsin(1 + t^2), \\ y(t) = \arccos^2 t \end{cases}$$

$$13.18. \begin{cases} x(t) = t + \sqrt{t^2 + 1}, \\ y(t) = \ln(t + \sqrt{t^2 + 1}) \end{cases}$$

$$13.32. \begin{cases} x(t) = \sqrt{t^3 + 2t^2 - 1}, \\ y(t) = \operatorname{arctg}(t - \sqrt{\sin t}) \end{cases}$$

$$13.33. \begin{cases} x(t) = \operatorname{ctg}(t + \ln t), \\ y(t) = \operatorname{tg}(1 + e^t) \end{cases}$$

$$13.34. \begin{cases} x(t) = t^2 + \cos(t^2 - 1), \\ y(t) = t - \sin t^2 \end{cases}$$

$$13.35. \begin{cases} x(t) = \operatorname{arctg}(2^t + 3^t), \\ y(t) = \lg(t^3 - 4) \end{cases}$$

$$13.36. \begin{cases} x(t) = \operatorname{arctg}(\lg t - \ln t), \\ y(t) = t^2 - 2^t \end{cases}$$

Задание № 14. Найти производную второго порядка от функции, заданной параметрически.

$$14.1. \begin{cases} x(t) = 2t - \cos t, \\ y(t) = 3 + \sin t \end{cases}$$

$$14.19. \begin{cases} x(t) = 2(1 + \sin t), \\ y(t) = 4(t - 2 \cos t) \end{cases}$$

$$14.2. \begin{cases} x(t) = \ln(t - 1), \\ y(t) = \sqrt{t^2 + 1} \end{cases}$$

$$14.20. \begin{cases} x(t) = \arcsin t, \\ y(t) = \frac{t^3}{3} \end{cases}$$

$$14.3. \begin{cases} x(t) = \sin^3 t, \\ y(t) = \operatorname{ctg}^2 t \end{cases}$$

$$14.21. \begin{cases} x(t) = \sin t, \\ y(t) = \cos^3 t \end{cases}$$

$$14.4. \begin{cases} x(t) = 2e^t, \\ y(t) = \arccos t \end{cases}$$

$$14.22. \begin{cases} x(t) = 3t - \cos t, \\ y(t) = 3 - \sin t \end{cases}$$

$$14.5. \begin{cases} x(t) = t^3, \\ y(t) = \frac{1}{t^3 + 4} \end{cases}$$

$$14.23. \begin{cases} x(t) = e^t(\cos t + \sin t), \\ y(t) = e^t(\sin t - \cos t) \end{cases}$$

$$14.6. \begin{cases} x(t) = \operatorname{tg}^2 t, \\ y(t) = \cos t \end{cases}$$

$$14.24. \begin{cases} x(t) = \sin 2t, \\ y(t) = 4 \cos^3 t \end{cases}$$

$$14.7. \begin{cases} x(t) = \sqrt[3]{\operatorname{ch} t}, \\ y(t) = \operatorname{sh} t \end{cases}$$

$$14.25. \begin{cases} x(t) = \frac{t^4}{4}, \\ y(t) = \ln \frac{t}{2} \end{cases}$$

$$14.8. \begin{cases} x(t) = \ln(3 \cos t), \\ y(t) = 2 \sin t \end{cases}$$

$$14.26. \begin{cases} x(t) = \operatorname{sh} t, \\ y(t) = \operatorname{th} t \end{cases}$$

$$14.9. \begin{cases} x(t) = \operatorname{ctg} t, \\ y(t) = \frac{1}{\cos 2t} \end{cases}$$

$$14.27. \begin{cases} x(t) = \sqrt{4 - t}, \\ y(t) = \ln(t + 5) \end{cases}$$

$$14.10. \begin{cases} x(t) = \sqrt[3]{t - 1}, \\ y(t) = \frac{2t}{\sqrt[3]{t - 1}} \end{cases}$$

$$14.28. \begin{cases} x(t) = e^t, \\ y(t) = \arccos 2t \end{cases}$$

$$14.11. \begin{cases} x(t) = \frac{1}{t^2}, \\ y(t) = \frac{1}{t^3 + 1} \end{cases}$$

$$14.29. \begin{cases} x(t) = \frac{2t}{t + 1}, \\ y(t) = \sqrt{t} \end{cases}$$

$$14.12. \begin{cases} x(t) = 3e^t, \\ y(t) = \operatorname{arctg} t \end{cases}$$

$$14.30. \begin{cases} x(t) = \sqrt[4]{t + 1}, \\ y(t) = \frac{1}{t} \end{cases}$$

$$14.13. \begin{cases} x(t) = t \cos t - \sin t, \\ y(t) = t \sin t + \cos t \end{cases}$$

$$14.31. \begin{cases} x(t) = \lg(2t - 1), \\ y(t) = \operatorname{sh} 2t \end{cases}$$

$$14.14. \begin{cases} x(t) = \sin 3t, \\ y(t) = 2(\cos 2t + \sin t) \end{cases}$$

$$14.32. \begin{cases} x(t) = \arcsin 4t, \\ y(t) = \frac{1}{t^2} \end{cases}$$

$$14.15. \begin{cases} x(t) = \sqrt[3]{1-t}, \\ y(t) = \frac{1}{t} \end{cases}$$

$$14.33. \begin{cases} x(t) = \frac{1}{\sqrt{t}}, \\ y(t) = \sqrt[3]{t^2 + 1} \end{cases}$$

$$14.16. \begin{cases} x(t) = \operatorname{sh}^2 t, \\ y(t) = \sqrt{\operatorname{ch} t} \end{cases}$$

$$14.34. \begin{cases} x(t) = \operatorname{ch}^3 t, \\ y(t) = \sqrt[3]{\operatorname{th} t} \end{cases}$$

$$14.17. \begin{cases} x(t) = \ln \operatorname{tg} t, \\ y(t) = \sin t \end{cases}$$

$$14.35. \begin{cases} x(t) = \sqrt[4]{t-1}, \\ y(t) = \frac{\lg t}{t} \end{cases}$$

$$14.18. \begin{cases} x(t) = \ln \operatorname{ctg} t, \\ y(t) = \cos t \end{cases}$$

$$14.36. \begin{cases} x(t) = \cos \lg 2t, \\ y(t) = \operatorname{tg} 3t \end{cases}$$

Задание № 15. Найти производную n -го порядка.

$$15.1. y = \cos(3x - 1) - \sin 2x$$

$$15.2. y = (x + 4)^{\frac{1}{5}}$$

$$15.3. y = \ln(2x + 3)$$

$$15.4. y = 2^{3x+1}$$

$$15.5. y = \sqrt{3^{4x-1}}$$

$$15.6. y = \sin(5x + 3) + 4 \cos(1 - x)$$

$$15.7. y = \frac{5}{x - 2}$$

$$15.8. y = 4^{3x-1}$$

$$15.9. y = \lg(4 - x)$$

$$15.10. y = \frac{x + 3}{x - 2}$$

$$15.11. y = \log_2(2x - 1)$$

$$15.12. y = \sin x \cdot \cos x$$

$$15.13. y = \frac{4 + x}{3(x + 1)}$$

$$15.14. y = 4 \sin kx + 2 \cos mx$$

$$15.15. y = a^{kx+3}$$

$$15.16. y = \sqrt[3]{2 - 3x}$$

$$15.17. y = \frac{1}{ax + b}$$

$$15.18. y = \frac{3x + 5}{9x + 12}$$

$$15.19. y = (ax^2 + bx + c)^5$$

$$15.20. y = \sqrt[12]{5^{1-x}}$$

$$15.21. y = \log_4(12x + 13)$$

$$15.22. y = 14^{3x}$$

$$15.23. y = (21 - x)^{\frac{4}{3}}$$

$$15.24. y = 5^{4-x}$$

$$15.25. y = \frac{2x - 1}{x + 2}$$

$$15.26. y = \sin x + e^{2x}$$

$$15.27. y = 5 \cos x + e^x$$

$$15.28. y = \log_2(17x - 2)$$

$$15.29. y = \frac{4}{3 - 2x}$$

$$15.30. y = \log_{11}(4x + 3)$$

$$15.31. y = \ln(x + 1) + e^x$$

$$15.32. y = \sin(x + 1) + \frac{2}{x}$$

$$15.33. y = \cos(2x - 1) + e^{2x-1}$$

$$15.34. y = \sqrt[7]{5^{4x-3}}$$

$$15.35. y = \frac{x}{x - 4}$$

$$15.36. y = \log_3(4x - 5)^2$$

Задание № 16. Найти производную неявной функции.

16.1. $xy^2 - x^2y = 0$

16.2. $\sin(xy) - xy = 0$

16.3. $\cos(1 - x^3y^3) = 0$

16.4. $\operatorname{tg} \frac{x+y}{x-y} = 2$

16.5. $\sin x(y-3) + \cos(xy) = 1$

16.6. $\frac{x-y^3 + \sin(xy)}{x^2y^2 + 4x} = 3$

16.7. $e^{xy} - xy^2 = 4$

16.8. $x^3y^2 - 2xy^2 + xy = 0$

16.9. $y \operatorname{tg} x - \sin^2(xy) = 0$

16.10. $x^3 - y^3 + 3x^2y = 0$

16.11. $y^3 - x^3 - 3xy^2 = 0$

16.12. $x^2 \sin(x-y)^2 = 1$

16.13. $\operatorname{tg}(x^2y) - \operatorname{ctg}(xy^2) = 2$

16.14. $\ln(xy - x^2y^3 + 1) = 4$

16.15. $\operatorname{arctg} \frac{x+y}{x-y} = 1$

16.16. $\arcsin \frac{x^2y}{(x-y)^2} - 2x^3y^3 = 1$

16.17. $\arccos \frac{1+xy}{e^{xy}} - xy = 2$

16.18. $\operatorname{arctg}(xy^2 - x^2y) - 3xy^3 = 0$

16.19. $\arcsin \frac{e^{x-y}}{x-y} - 2y^2 = 0$

16.20. $\operatorname{arctg} \frac{\sqrt{x-y}}{1-\sqrt{y}} - y = 0$

16.21. $\frac{e^{x-3y}}{x-y} = 1$

16.22. $\ln \frac{y+6x}{y} - 1 = 0$

16.23. $e^{\sqrt{2}\cos y} - \cos(xy) = 0$

16.24. $\sqrt[3]{\sin(xy^2) - \cos(x^2y)} = 1$

16.25. $\ln(\cos x - \cos y) = 2$

16.26. $\sqrt[5]{\operatorname{tg}(x-y) + \operatorname{ctg}(x+y)} = 1$

16.27. $e^{\sqrt{y}\sin(x-y)-x^2y} = 1$

16.28. $\ln(\sqrt{xy} + x^3y^3) - 2xy = 0$

16.29. $(x+y)(x-y)(x^2-y^2) = 1$

16.30. $\sin(x^2-y^2) + \sqrt[3]{x^3-y^3} = 0$

16.31. $y \sin(xy+x) + 2y \operatorname{tg}(x+y) = 3x$

16.32. $\frac{xy^2 - x^3y + 2xy}{\operatorname{tg}(x^2y)} = x^2$

16.33. $\ln(x+y) \sin(x-2y) = xy$

16.34. $\operatorname{tg}(x^2+y^2-1) = \ln(x+y-1)$

16.35. $e^{xy+\sin(xy)} = x-y$

16.36. $\sqrt{\sin x + \operatorname{tg}(x^2+y^2)} = y$

Раздел 2.

ГРАФИКИ ФУНКЦИЙ

Задание № 1. Построить графики функций, используя при их исследовании первую производную.

1.1. $f(x) = 2x^3 - 3x^2 - 12x + 6$

1.2. $f(x) = x^4 - 4x^3 + 16x - 4$

1.3. $f(x) = -x(x + 3)^2$

1.4. $f(x) = (x^2 - 1)^2$

1.5. $f(x) = 2x^3 + 3x^2 - 12x + 1$

1.6. $f(x) = x^4 - 6x^3 + 8x + 4$

1.7. $f(x) = (x + 2)^2(x - 3)^2$

1.8. $f(x) = -2x(x - 5)^2$

1.9. $f(x) = 2x^3 + 9x^3 - 3$

1.10. $f(x) = x^4 - 4x^3 + 5$

1.11. $f(x) = 3x^2(x + 4)^2$

1.12. $f(x) = 5x(x - 2)^2$

1.13. $f(x) = 3x^4 - 4x^3 - 12x^2 - 4$

1.14. $f(x) = -x^2(x - 1)^2$

1.15. $f(x) = 4x(x - 4)^2$

1.16. $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$

1.17. $f(x) = x^3 - 3x^2 - 24x + 3$

1.18. $f(x) = (2x + 3)^2(2x - 5)^2$

1.19. $f(x) = 3x^4 + 4x^3 - 12x^2 - 4$

1.20. $f(x) = x^4 - 8x^3 + 128x - 1$

1.21. $f(x) = (3x + 1)^2(x - 2)^2$

1.22. $f(x) = x^4 - 24x^2 - 64x + 1$

1.23. $f(x) = -x^2(x + 3)^2$

1.24. $f(x) = x^3 + 3x^2 - 24x - 4$

1.25. $f(x) = 3x^4 + 8x^3 - 48x^2 + 4$

1.26. $f(x) = x^4 + 8x^3 + 22x^2 + 24x - 3$

1.27. $f(x) = 1 + 15 + 6x^2 - x^3$

1.28. $f(x) = x^4 - 14x^2 - 24x + 5$

1.29. $f(x) = -(x + 1)^2(2x - 3)^2$

1.30. $f(x) = 7 + 72x + 3x^2 - 2x^2$

1.31. $f(x) = x^3 - 3x + 2$

1.32. $f(x) = (x - 2)^2(x + 1)^2$

1.33. $f(x) = -(x - 1)^2(2x + 1)^2$

1.34. $f(x) = x^3 + 3x^2 - 4$

1.35. $f(x) = 2x(2x - 1)^2$

1.36. $f(x) = x^3 - 6x^2 + 5$

Задание № 2. Построить графики функций, используя при их исследовании первую производную.

2.1. $y = 2 - \sqrt[3]{x^2 + x - 2}$

2.19. $y = \sqrt[3]{x(x + 7)}$

2.2. $y = 2x + 1 - \sqrt[3]{(x - 1)^2}$

2.20. $y = x + \sqrt[3]{(x - 4)^2}$

2.3. $y = (3x + 2)\sqrt[3]{x^2}$

2.21. $y = 3 + (x + 1)\sqrt[3]{x^2}$

2.4. $y = \sqrt[3]{x^2} + \sqrt[3]{(x + 5)^2}$

2.22. $y = 4\sqrt[3]{x^2} + 3\sqrt[3]{(x - 1)^2}$

2.5. $y = -5 + \sqrt[3]{x(x - 2)}$

2.23. $y = \sqrt[3]{x^2 - x - 12}$

2.6. $y = 4 + \sqrt[3]{x^2 + 4x}$

2.24. $y = 2x + 3 + \sqrt[3]{(x + 5)^2}$

2.7. $y = x + 2 + 4\sqrt[3]{x^2}$

2.25. $y = 1 - 2(x - 3)\sqrt[3]{x^2}$

2.8. $y = (x - 4)\sqrt[3]{x^2}$

2.26. $y = 5\sqrt[3]{(x - 2)^2} + \sqrt[3]{x^2}$

2.9. $y = 2\sqrt[3]{(x - 1)^2} + \sqrt[3]{x^2}$

2.27. $y = 7x - 2 - \sqrt[3]{(x + 3)^2}$

2.10. $y = 1 - \sqrt[3]{(x + 1)(x - 3)}$

2.28. $y = 2x - \sqrt[3]{(x - 5)^2}$

2.11. $y = 3x + 5 - \sqrt[3]{(x + 2)^2}$

2.29. $y = 2\sqrt[3]{(x + 1)^2} - 3x$

2.12. $y = (2x - 5)\sqrt[3]{x^2}$

2.30. $y = 4\sqrt[3]{x^2} + \sqrt[3]{(x - 6)^2}$

2.13. $y = \sqrt[3]{(x - 3)^2} + \sqrt[3]{x^2}$

2.31. $y = 4x + 1 - \sqrt[3]{(x - 2)^2}$

2.14. $y = 3\sqrt[3]{x^2} + 8x - 2$

2.32. $y = 1 - x\sqrt[3]{(x - 1)^2}$

2.15. $y = 3 + \sqrt[3]{x^2 - 3x - 10}$

2.33. $y = 2\sqrt[3]{(x - 1)^2} + \sqrt[3]{(x + 2)^2}$

2.16. $y = 5x - 1 + 2\sqrt[3]{(x - 3)^2}$

2.34. $y = 2x - 1 + \sqrt[3]{(x + 1)^2}$

2.17. $y = (3 - 2x)\sqrt[3]{x^2}$

2.35. $y = 2 + (x + 2)\sqrt[3]{x^2}$

2.18. $y = 3\sqrt[3]{x^2} + \sqrt[3]{(x + 2)^2}$

2.36. $y = 2\sqrt[3]{-x^2} - \sqrt[3]{(x + 3)^2}$

Задание № 3. Найти наибольшее и наименьшее значение функции на заданном отрезке .

$$3.1. \quad y = x - \ln(1 + x); \quad x \in \left[-\frac{1}{2}; 1\right]$$

$$3.2. \quad y = \frac{1 + 3x}{\sqrt{4 + 5x^2}}; \quad x \in [0; 3]$$

$$3.3. \quad y = \frac{1}{x\sqrt{1-x}}; \quad x \in \left[\frac{1}{10}; \frac{3}{4}\right]$$

$$3.4. \quad y = (x - 2)^5(2x + 1)^4; \quad x \in [-1; 1]$$

$$3.5. \quad y = x\sqrt{ax - x^2}; \quad x \in [0; a]$$

$$3.6. \quad y = 2 \sin x + \cos 2x; \quad x \in \left[0; \frac{\pi}{2}\right]$$

$$3.7. \quad y = 2x^2 - \ln x; \quad x \in \left[\frac{1}{10}; 2\right]$$

$$3.8. \quad y = \frac{10}{4x^3 - 9x^2 + 6x + 1}; \quad x \in [0; 2]$$

$$3.9. \quad y = \frac{3x^2 + 4x + 4}{x^2 + x + 1}; \quad x \in [-1; 1]$$

$$3.10. \quad y = \ln(x^4 + 4x^3 + 30); \quad x \in [-4; 1]$$

$$3.11. \quad y = x \sin x + \cos x - \frac{x^2}{4}; \quad x \in \left[0; \frac{\pi}{2}\right]$$

$$3.12. \quad y = (x^2 - 2x) \ln x - \frac{3x^2}{2} + 4x; \quad x \in \left[\frac{1}{2}; 3\right]$$

$$3.13. \quad y = (x - 5)^2 \sqrt[3]{(x + 1)^2}; \quad x \in [0; 6]$$

$$3.14. \quad y = x - \ln(1 + x^2); \quad x \in [0; 2]$$

$$3.15. \quad y = \frac{2}{3} x^2 \sqrt[3]{6x - 7}; \quad x \in [-1; 2]$$

$$3.16. \quad y = -x^2 \sqrt{x^2 + 2}; \quad x \in [-1; 2]$$

$$3.17. \quad y = \sqrt[3]{x^3 - 3x^2 + 64}; \quad x \in [0; 4]$$

$$3.18. \quad y = \ln(x + \sqrt{x^2 + 4}); \quad x \in [1; 4]$$

$$3.19. \quad y = \frac{1 - x + x^2}{1 + x - x^2}; \quad x \in [0; 1]$$

$$3.20. \quad y = x^5 - 5x^4 + 5x^3 + 1; \quad x \in [-1; 2]$$

$$3.21. \quad y = \sqrt{e^{2x} + e^{-x}}; \quad x \in [0; 1]$$

$$3.22. \quad y = \arccos x^2; \quad x \in \left[-\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right]$$

$$3.23. \quad y = 2x^2 - \frac{4}{x} + 3; \quad x \in \left[-2; -\frac{1}{2}\right]$$

$$3.24. \quad y = \sqrt{x(2-x)}; \quad x \in [0; 2]$$

$$3.25. \quad y = \sin 2x - x; \quad x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$$

$$3.26. \quad y = 2 \operatorname{tg} x - \operatorname{tg}^2 x; \quad x \in \left[0; \frac{\pi}{3}\right]$$

$$3.27. \quad y = \sqrt[3]{(x^2 - 2x)^2}; \quad x \in [0; 3]$$

$$3.28. \quad y = \operatorname{arctg} \frac{1-x}{1+x}; \quad x \in [0; 1]$$

$$3.29. \quad y = x^2 + \frac{2}{x} - 4; \quad x \in \left[\frac{1}{2}; 3\right]$$

$$3.30. \quad y = \cos 2x + 2x, \quad x \in [-\pi; \pi]$$

$$3.31. \quad y = |x^2 - 6x + 5|; \quad x \in [0; 6]$$

$$3.32. \quad y = \sqrt[3]{x^3 - 3x + 3}; \quad x \in [-1; 2]$$

$$3.33. \quad y = \frac{x^2 - 3x + 2}{(1+x)^2}; \quad x \in [0; 3]$$

$$3.34. \quad y = \frac{x}{x^2 + 2x + 4} + 3; \quad x \in [1; 4]$$

$$3.35. \quad y = |3 + 4x + x^2|; \quad x \in [-3; 0]$$

$$3.36. \quad y = \frac{1}{\sqrt[4]{x^4 - 4x^2 + 5}}; \quad x \in [-1; 2]$$

Задание № 4.

Варианты 1 – 6. В основании прямоугольного параллелепипеда лежит прямоугольник, отношение длин сторон которого равно k . Площадь полной поверхности параллелепипеда равна S . При заданных k и S найти максимально возможный объём такого параллелепипеда.

4.1	$k = 4/3$	$S = 20$	4.2	$k = 2/3$	$S = 50$	4.3	$k = 2$	$S = 30$
4.4	$k = 5/3$	$S = 100$	4.5	$k = 2/5$	$S = 150$	4.6	$k = 7/3$	$S = 20$

Варианты 7 – 12. Сечение тоннеля имеет форму прямоугольника, завершённого полукругом. Периметр сечения равен p . При каком радиусе полукруга площадь сечения будет наибольшей?

4.7	$p = 20$	4.8	$p = 35$	4.9	$p = 25$	4.10	$p = 22$	4.11	$p = 37$	4.12	$p = 50$
-----	----------	-----	----------	-----	----------	------	----------	------	----------	------	----------

Варианты 13 – 18. Фигура на плоскости ограничена линиями $y = 0$ и $y = \sqrt[3]{(x-a)(b-x)}$. В эту фигуру вписан прямоугольник, основание которого лежит на прямой $y = 0$. При заданных a и b найти максимально возможную площадь такого прямоугольника.

4.13	$a = -2$	$b = 4$	4.14	$a = -1$	$b = 3$	4.15	$a = 0$	$b = 8$
4.16	$a = -3$	$b = 5$	4.17	$a = 1$	$b = 5$	4.18	$a = 2$	$b = 4$

Варианты 19 – 24. В основании прямоугольного параллелепипеда лежит прямоугольник, отношение длин сторон которого равно k . Сумма площади боковой поверхности параллелепипеда и площади нижнего основания равна S . При заданных k и S найти максимально возможный объём такого параллелепипеда.

4.19	$k = 3/5$	$S = 20$	4.20	$k = 2/5$	$S = 50$	4.21	$k = 6/5$	$S = 30$
4.22	$k = 5/7$	$S = 100$	4.23	$k = 3/7$	$S = 150$	4.24	$k = 7/4$	$S = 20$

Варианты 25 – 30. В основании прямой призмы лежит равнобедренный треугольник, отношение длины боковой стороны которого к длине основания равно k . Площадь полной поверхности призмы равна S . При заданных k и S найти максимально возможный объём такой призмы.

4.25	$k = 4/3$	$S = 20$	4.26	$k = 2/3$	$S = 50$	4.27	$k = 6/5$	$S = 30$
4.28	$k = 5/3$	$S = 100$	4.29	$k = 5/2$	$S = 150$	4.30	$k = 7/3$	$S = 20$

Варианты 31 – 36. Геометрическое тело представляет из себя объединение прямого цилиндра и полушария, причём верхнее основание цилиндра совпадает с нижним основанием полушария. Объём тела равен V . Какова минимально возможная площадь поверхности этого тела?

4.31	$V = 45\pi$	4.32	$V = 90\pi$	4.33	$V = 135\pi$
4.34	$V = 180\pi$	4.35	$V = 405\pi$	4.36	$V = 270\pi$



Задание № 5. Провести полное исследование функции и построить её график.

5.1. $y = x(1 - x^2)$

5.19. $y = x(x^2 - 1) - 6$

5.2. $y = x^2(1 - x^2)$

5.20. $y = x^2(x^2 - 1) + 1$

5.3. $y = x(x^2 - 1)$

5.21. $y = (x^2 - 1)^2 - 1$

5.4. $y = x^2(x^2 - 1)$

5.22. $y = 1 - x^2(1 - x^2)$

5.5. $y = x(1 - x)^2$

5.23. $y = 6 - x(x^2 - 1)$

5.6. $y = x^2(1 - x)^2$

5.24. $y = 3 - x(x^2 - 4)$

5.7. $y = (x^2 - 4)^2$

5.25. $y = 1 - (x^2 - 9)^2$

5.8. $y = (x - 1)^2(x + 1)$

5.26. $y = (1 - x)(x + 1)^2 - 1$

5.9. $y = (x - 1)(x + 1)^2$

5.27. $y = 9 + (1 - x)(x + 1)^2$

5.10. $y = -(x - 1)^2(x + 1)$

5.28. $y = (x - 2)^2(x + 1) - 2$

5.11. $y = (2 - x)(x + 1)^2$

5.29. $y = x(x^2 - 3x + 2)$

5.12. $y = (x - 2)^2(x + 1)$

5.30. $y = x^3 + 3x^2 + 2x$

5.13. $y = (x - 3)(x + 1)^2$

5.31. $y = x^3 - 3x + 2$

5.14. $y = 4x - x^3$

5.32. $y = x(2 - x^2)$

5.15. $y = 4x^3 - x$

5.33. $y = x^3 - 6x + 5$

5.16. $y = 4x^2 - x^3$

5.34. $y = (x + 2)^2(x - 2)$

5.17. $y = 4x^3 - x^2$

5.35. $y = x^3 - 6x + 4$

5.18. $y = x^2(1 - x^2) - 1$

5.36. $y = x^2(3 - x)$

Задание № 6. Провести полное исследование функции и построить её график.

6.1. $y = \sqrt[3]{x(x-1)}$

6.19. $y = \sqrt[3]{(x-3)^2}$

6.2. $y = \sqrt[3]{(x-1)^2} - 1$

6.20. $y = \sqrt[3]{(1-x)(x+1)}$

6.3. $y = 1 - \sqrt[3]{x(x-1)}$

6.21. $y = \sqrt[3]{(1-x)(x+3)} - 1$

6.4. $y = \sqrt[3]{(x-2)^2}$

6.22. $y = 1 - \sqrt[3]{x(3-x)}$

6.5. $y = 1 - \sqrt[3]{(x-2)^2}$

6.23. $y = 1 - \sqrt[3]{(x+3)^2}$

6.6. $y = \sqrt[3]{x(x+1)}$

6.24. $y = 1 - \sqrt[3]{(4-x)(x+1)}$

6.7. $y = \sqrt[3]{(x+2)^2} - 1$

6.25. $y = \sqrt[3]{(2-x)(x+1)}$

6.8. $y = \sqrt[3]{x(x+1)} - 1$

6.26. $y = \sqrt[3]{(2-x)(x+3)} - 1$

6.9. $y = 1 - \sqrt[3]{x(x+2)}$

6.27. $y = \sqrt[3]{(2x-1)^2}$

6.10. $y = \sqrt[3]{(x+1)(x-1)}$

6.28. $y = \sqrt[3]{(2-x)(x+4)}$

6.11. $y = \sqrt[3]{(x+2)(x-1)}$

6.29. $y = \sqrt[3]{(x-3)(x-1)}$

6.12. $y = 1 - \sqrt[3]{(x-3)(x+1)}$

6.30. $y = -\sqrt[3]{(2x+1)^2}$

6.13. $y = \sqrt[3]{(x+1)(x-2)}$

6.31. $y = \sqrt[3]{(x-2)(x-3)} - 1$

6.14. $y = \sqrt[3]{(x+4)(x-2)} - 1$

6.32. $y = 1 - \sqrt[3]{(3-x)(5-x)}$

6.15. $y = 1 - \sqrt[3]{(x+3)(x-2)}$

6.33. $y = \sqrt[3]{(3-x)(x-4)}$

6.16. $y = \sqrt[3]{(x+1)^2} - 1$

6.34. $y = \sqrt[3]{(4-x)^2} - 1$

6.17. $y = \sqrt[3]{x(1-x)}$

6.35. $y = \sqrt[3]{(3-x)(x+3)}$

6.18. $y = \sqrt[3]{x(2-x)} - 1$

6.36. $y = \sqrt[3]{x^2 - 4} - 1$

Задание № 7. Провести полное исследование функции и построить её график.

7.1. $y = \frac{x^2 - 2x}{x + 1}$

7.2. $y = \frac{x^2 + 2x}{x + 1}$

7.3. $y = \frac{x^2 - 2x}{x - 1}$

7.4. $y = \frac{x^2 + 2x}{x - 1}$

7.5. $y = \frac{2x - x^2}{x + 1}$

7.6. $y = \frac{2x - x^2}{x - 1}$

7.7. $y = \frac{(x + 1)^2}{x - 1}$

7.8. $y = \frac{(x - 1)^2}{x + 1}$

7.9. $y = \frac{(x + 1)^2}{2 - x}$

7.10. $y = -\frac{(x - 1)^2}{x + 3}$

7.11. $y = \frac{x^2 - 1}{x + 2}$

7.12. $y = \frac{(2 - x)(x + 1)}{x + 2}$

7.13. $y = \frac{x^2 - 1}{x - 2}$

7.14. $y = \frac{(x - 3)(x + 1)}{2 - x}$

7.15. $y = \frac{2x^2 - 2}{x - 2}$

7.16. $y = \frac{6 - 2x^2}{x - 2}$

7.17. $y = \frac{2x^2 - 2}{4 - x}$

7.18. $y = \frac{2x^2 + 2}{x + 2}$

7.19. $y = \frac{2x^2 + 2}{x - 2}$

7.20. $y = \frac{2x^2 - 2}{x + 2}$

7.21. $y = \frac{x^2 + 2x + 2}{x + 1}$

7.22. $y = \frac{x^2 - 2x + 2}{x - 1}$

7.23. $y = \frac{(x + 1)^2 - 3}{x + 1}$

7.24. $y = \frac{(x - 1)^2 - 5}{x - 1}$

7.25. $y = \frac{4 - (x + 1)^2}{x + 1}$

7.26. $y = \frac{2 - (x - 1)^2}{x - 1}$

7.27. $y = \frac{x^2 + x}{x - 1}$

7.28. $y = \frac{x^2 - x}{x + 1}$

7.29. $y = \frac{2x - x^2}{3 - x}$

7.30. $y = \frac{x^2 - x}{2 - x}$

7.31. $y = \frac{x^2 + x - 2}{x + 3}$

7.32. $y = \frac{2x^2 - 3x - 1}{x - 3}$

7.33. $y = \frac{(x + 1)^2}{x^2}$

7.34. $y = \frac{x^2}{(x - 1)^2}$

7.35. $y = \frac{(x - 1)^2}{x^2}$

7.36. $y = \frac{x^2}{(x + 1)^2}$

Задание № 8. Провести полное исследование функции и построить её график.

8.1. $y = \frac{e^{x-1}}{x-1}$

8.19. $y = \frac{e^{4-2x}}{4-x}$

8.2. $y = \frac{x^2 + 2x}{x+1}$

8.20. $y = (2x-1)e^{2-x}$

8.3. $y = \frac{e^{x-1}}{2-x}$

8.21. $y = (2x-1)e^{x+2}$

8.4. $y = \frac{e^{1-x}}{2-x}$

8.22. $y = (2x+1)e^{2-x}$

8.5. $y = (x-1)e^{x-1}$

8.23. $y = (2x+1)e^{2+x}$

8.6. $y = (2-x)e^{x-1}$

8.24. $y = \frac{e^{x+2}}{x+1}$

8.7. $y = (x-1)e^{1-x}$

8.25. $y = \frac{e^{x-2}}{x+1}$

8.8. $y = (2-x)e^{1-x}$

8.26. $y = \frac{e^{x+2}}{x-1}$

8.9. $y = \frac{e^{x+1}}{x-1}$

8.27. $y = \frac{e^{x-2}}{x-1}$

8.10. $y = \frac{e^{x-1}}{2-x}$

8.28. $y = (1+x)e^{x+2}$

8.11. $y = \frac{e^{-x-1}}{x-1}$

8.29. $y = (x-1)e^{x+2}$

8.12. $y = \frac{e^{-x-1}}{1-2x}$

8.30. $y = (x-3)e^{x-2}$

8.13. $y = (x-1)e^{x+1}$

8.31. $y = \frac{e^{2x-1}}{x}$

8.14. $y = (2-x)e^{x+1}$

8.32. $y = xe^{1-x}$

8.15. $y = (x-1)e^{-x-1}$

8.33. $y = \frac{e^{2-x}}{1-x}$

8.16. $y = (2-x)e^{-x-1}$

8.34. $y = (x-1)e^{3-x}$

8.17. $y = \frac{e^{x+2}}{2x-4}$

8.35. $y = \frac{e^{2x+1}}{x-2}$

8.18. $y = \frac{e^{2-x}}{2x-4}$

8.36. $y = (x+1)e^{-2x}$

Задание № 9. Провести полное исследование функции и построить её график.

9.1. $y = e^{\sin x + \cos x}$

9.19. $y = \operatorname{arctg}(\sin x)$

9.2. $y = e^{-\sin x - \cos x}$

9.20. $y = \operatorname{arctg}(\cos x)$

9.3. $y = e^{\cos x - \sin x}$

9.21. $y = \operatorname{arctg}(\sin x + \cos x)$

9.4. $y = e^{-\sin 2x}$

9.22. $y = \operatorname{arctg}(\sin x - \cos x)$

9.5. $y = e^{\sin 2x}$

9.23. $y = \ln \operatorname{tg} x$

9.6. $y = e^{\operatorname{tg} x}$

9.24. $y = \ln \operatorname{arctg} x$

9.7. $y = e^{-\operatorname{tg} x}$

9.25. $y = e^{2 \sin^2 x}$

9.8. $y = e^{\operatorname{ctg} x}$

9.26. $y = e^{2 \cos^2 x}$

9.9. $y = e^{-\operatorname{ctg} x}$

9.27. $y = e^{|\sin x|}$

9.10. $y = e^{\operatorname{arctg} x}$

9.28. $y = e^{|\cos x|}$

9.11. $y = \ln \sin x$

9.29. $y = e^{-|\sin x|}$

9.12. $y = \ln(-\sin x)$

9.30. $y = e^{-|\cos x|}$

9.13. $y = \ln(\sin x + \cos x)$

9.31. $y = \ln(x^2 + 2x + 2)$

9.14. $y = \ln(\sin x - \cos x)$

9.32. $y = \ln(1 + 3x^4)$

9.15. $y = \ln \cos x$

9.33. $y = x^x$

9.16. $y = \ln(-\cos x)$

9.34. $y = \ln(x^2 - 6x + 10)$

9.17. $y = \operatorname{arctg}(\ln x)$

9.35. $y = e^{-(x-1)^2}$

9.18. $y = \operatorname{arctg}(\ln x)$

9.36. $y = x^{2x}$

Задание № 10. Провести полное исследование функции и построить её график.

- | | |
|----------------------------------|--------------------------------------|
| 10.1. $y = x^2 \ln x$ | 10.19. $y = \frac{\ln(-x)}{x}$ |
| 10.2. $y = x^2 \ln(-x)$ | 10.20. $y = x^3 \ln x$ |
| 10.3. $y = x \ln x$ | 10.21. $y = x^3 \ln(-x)$ |
| 10.4. $y = x \ln(-x)$ | 10.22. $y = x^3 \ln^2 x$ |
| 10.5. $y = x \ln^2 x$ | 10.23. $y = x^3 \ln^2(-x)$ |
| 10.6. $y = x \ln^2(-x)$ | 10.24. $y = \frac{x^3}{\ln x}$ |
| 10.7. $y = x^2 \ln^2 x$ | 10.25. $y = \frac{x^3}{\ln^2 x}$ |
| 10.8. $y = x^2 \ln^2(-x)$ | 10.26. $y = \frac{x^3}{\ln(-x)}$ |
| 10.9. $y = \frac{x}{\ln x}$ | 10.27. $y = \frac{x^3}{\ln^2(-x)}$ |
| 10.10. $y = \frac{x^2}{\ln x}$ | 10.28. $y = \frac{\ln x}{x^3}$ |
| 10.11. $y = \frac{x^2}{\ln^2 x}$ | 10.29. $y = \frac{\ln^2 x}{x^3}$ |
| 10.12. $y = \frac{x}{\ln^2 x}$ | 10.30. $y = \frac{\ln^2(-x)}{x^3}$ |
| 10.13. $y = \frac{x}{\ln(-x)}$ | 10.31. $y = \frac{x^2}{\ln^2(-x)}$ |
| 10.14. $y = \frac{x}{\ln^2(-x)}$ | 10.32. $y = \frac{\ln^2(-x)}{x^2}$ |
| 10.15. $y = \frac{\ln x}{x}$ | 10.33. $y = x^4 \ln x$ |
| 10.16. $y = \frac{\ln x}{x^2}$ | 10.34. $y = x^4 \ln(-x)$ |
| 10.17. $y = \frac{\ln^2 x}{x}$ | 10.35. $y = \sqrt{x} \ln x$ |
| 10.18. $y = \frac{\ln^2 x}{x^2}$ | 10.36. $y = \frac{\ln x}{x\sqrt{x}}$ |

Задание № 11. Провести полное исследование функции и построить её график.

- | | |
|----------------------------|----------------------------|
| 11.1. $y = x + e^x$ | 11.19. $y = 3x + 2e^{-x}$ |
| 11.2. $y = -x + e^x$ | 11.20. $y = 3x - 2e^x$ |
| 11.3. $y = x + e^{-x}$ | 11.21. $y = -3x + 2e^{-x}$ |
| 11.4. $y = x - e^x$ | 11.22. $y = -3x - 2e^x$ |
| 11.5. $y = -x + e^{-x}$ | 11.23. $y = x + 3e^{-2x}$ |
| 11.6. $y = x - e^{-x}$ | 11.24. $y = x + 3e^{2x}$ |
| 11.7. $y = 2x - e^x$ | 11.25. $y = x - 3e^{2x}$ |
| 11.8. $y = 2x + e^{-x}$ | 11.26. $y = x - 3e^{-2x}$ |
| 11.9. $y = 2x + e^{-2x}$ | 11.27. $y = -x + 3e^{2x}$ |
| 11.10. $y = -2x - e^x$ | 11.28. $y = -x - 3e^{2x}$ |
| 11.11. $y = x - 2e^{-2x}$ | 11.29. $y = -2x + 3e^{-x}$ |
| 11.12. $y = x + 2e^{-2x}$ | 11.30. $y = 2x + 3e^x$ |
| 11.13. $y = x + 2e^{2x}$ | 11.31. $y = -x + 2e^{-x}$ |
| 11.14. $y = -x - 2e^{-2x}$ | 11.32. $y = -x - 2e^x$ |
| 11.15. $y = 3x + e^{2x}$ | 11.33. $y = -x - e^{3x}$ |
| 11.16. $y = -3x + e^{2x}$ | 11.34. $y = x - e^{3x}$ |
| 11.17. $y = 3x + e^{-2x}$ | 11.35. $y = x + e^{3x}$ |
| 11.18. $y = 3x - e^{-2x}$ | 11.36. $y = -x - e^{-3x}$ |

Задание № 12. Построить графики функций в полярных координатах.

12.1. $\rho = a \sin 2\varphi$

12.2. $\rho = a \cos 2\varphi$

12.3. $\rho = a \sin 3\varphi$

12.4. $\rho = a \cos 3\varphi$

12.5. $\rho = a \sin 4\varphi$

12.6. $\rho = a \cos 4\varphi$

12.7. $\rho = a \operatorname{tg} \varphi$

12.8. $\rho = a(1 + \operatorname{tg} \varphi)$

12.9. $\rho = 1 + \cos \varphi$

12.10. $\rho = 2 + \cos \varphi$

12.11. $\rho = 1 + 2 \cos \varphi$

12.12. $\rho = 1 + \sin \varphi$

12.13. $\rho = 1 + 2 \sin \varphi$

12.14. $\rho = 2 + \sin \varphi$

12.15. $\rho = 1 + 3 \cos \varphi$

12.16. $\rho = 2(1 + 3 \cos \varphi)$

12.17. $\rho = 1 + 3 \sin \varphi$

12.18. $\rho = 2 - 3 \sin \varphi$

12.19. $\rho = 2(1 + 3 \sin \varphi)$

12.20. $(x^2 + y^2)^2 = 2ax^3$

12.21. $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$

12.22. $(x^2 + y^2)^3 = 4a^2x^2y^2$

12.23. $(x^2 + y^2)x = a^2y$

12.24. $x^4 + y^4 = a^2(x^2 + y^2)$

12.25. $x^4 - y^4 = 4x^2y^2$

12.26. $(x^2 + y^2)^3 = 4a^2xy(x^2 - y^2)$

12.27. $\rho = 2 - \cos 4\varphi$

12.28. $\rho = 3 - 2 \cos \varphi$

12.29. $\rho = 4 - \cos 2\varphi$

12.30. $\rho = 3 + \cos \varphi$

12.31. $\rho = 2(1 + \sin 2\varphi)$

12.32. $(x^2 + y^2)^2 = 2a^2x$

12.33. $\rho = 2 - \cos 3\varphi$

12.34. $8(x^2 + y^2)^3 = x^2y^2$

12.35. $(x^2 + y^2)^2 = 4ay^3$

12.36. $\rho = 1 - 2 \cos 2\varphi$

Задание № 13. Построить график функции, заданной параметрически.

$$13.1. \begin{cases} x(t) = 1 + \cos t, \\ y(t) = -1 + 4 \sin t \end{cases}$$

$$13.2. \begin{cases} x(t) = 1 + 2 \cos t, \\ y(t) = 1 + 2 \sin t \end{cases}$$

$$13.3. \begin{cases} x(t) = t^2 - 2t, \\ y(t) = t^2 + 2t \end{cases}$$

$$13.4. \begin{cases} x(t) = 3t, \\ y(t) = 6t - t^2 \end{cases}$$

$$13.5. \begin{cases} x(t) = \cos t, \\ y(t) = \sin 2t \end{cases}$$

$$13.6. \begin{cases} x(t) = t^3 + 1, \\ y(t) = t^2 \end{cases}$$

$$13.7. \begin{cases} x(t) = t - \sin t, \\ y(t) = 1 - \cos t \end{cases}$$

$$13.8. \begin{cases} x(t) = 1 - t^2, \\ y(t) = t - t^3 \end{cases}$$

$$13.9. \begin{cases} x(t) = \frac{t+1}{t}, \\ y(t) = \frac{t-1}{t} \end{cases}$$

$$13.10. \begin{cases} x(t) = \cos t, \\ y(t) = t + 2 \sin t \end{cases}$$

$$13.11. \begin{cases} x(t) = a \cos^3 t, \\ y(t) = a \sin^3 t \end{cases}$$

$$13.12. \begin{cases} x(t) = a \cos^5 t, \\ y(t) = a \sin^5 t \end{cases}$$

$$13.13. \begin{cases} x(t) = t^2 + 2t, \\ y(t) = t^3 + t \end{cases}$$

$$13.19. \begin{cases} x(t) = \ln t, \\ y(t) = t^3 \end{cases}$$

$$13.20. \begin{cases} x(t) = 2t, \\ y(t) = 3t^2 - 5t \end{cases}$$

$$13.21. \begin{cases} x(t) = t^3 + 2, \\ y(t) = 0.5t^2 \end{cases}$$

$$13.22. \begin{cases} x(t) = \frac{1}{t+2}, \\ y(t) = \left(\frac{t}{t+1}\right)^2 \end{cases}$$

$$13.23. \begin{cases} x(t) = 2^{-t}, \\ y(t) = 2^{2t} \end{cases}$$

$$13.24. \begin{cases} x(t) = \sqrt[3]{1 - \sqrt{t}}, \\ y(t) = \sqrt{1 - \sqrt[3]{t}} \end{cases}$$

$$13.25. \begin{cases} x(t) = te^t, \\ y(t) = te^{-t} \end{cases}$$

$$13.26. \begin{cases} x(t) = t - t^2, \\ y(t) = t + t^2 \end{cases}$$

$$13.27. \begin{cases} x(t) = t - \sin 2t, \\ y(t) = 1 - \cos 2t \end{cases}$$

$$13.28. \begin{cases} x(t) = t(3 - t^2), \\ y(t) = t^2 \end{cases}$$

$$13.29. \begin{cases} x(t) = t^2 + 1, \\ y(t) = t^3 - 3t \end{cases}$$

$$13.30. \begin{cases} x(t) = 2t - t^2, \\ y(t) = 2t - t^3 \end{cases}$$

$$13.31. \begin{cases} x(t) = t^3 + 1, \\ y(t) = t^2 - 2t + 1 \end{cases}$$

$$13.14. \begin{cases} x(t) = \sin t, \\ y(t) = \cos 2t \end{cases}$$

$$13.15. \begin{cases} x(t) = \frac{1}{t}, \\ y(t) = \frac{t}{t+1} \end{cases}$$

$$13.16. \begin{cases} x(t) = 2t - 1, \\ y(t) = t^3 \end{cases}$$

$$13.17. \begin{cases} x(t) = \sqrt{t}, \\ y(t) = \sqrt[3]{t} \end{cases}$$

$$13.18. \begin{cases} x(t) = e^{-t}, \\ y(t) = e^{2t} \end{cases}$$

$$13.32. \begin{cases} x(t) = 2t^3 - t, \\ y(t) = t^2 + t \end{cases}$$

$$13.33. \begin{cases} x(t) = 1 - \sin t, \\ y(t) = \cos t \end{cases}$$

$$13.34. \begin{cases} x(t) = 2t - t^3, \\ y(t) = t + 3 \end{cases}$$

$$13.35. \begin{cases} x(t) = \ln 2t, \\ y(t) = t^3 + t \end{cases}$$

$$13.36. \begin{cases} x(t) = t^2 + 2, \\ y(t) = t^2 - 2t \end{cases}$$