

## 5. Изопериметрические задачи

5.1. Найдите экстремали в следующих задачах

а)  $\int_0^1 \dot{x}^2 dt \rightarrow \min, \int_0^1 x dt = 0, x(0) = 0, x(1) = 1;$

б)  $\int_0^1 \dot{x}^2 dt \rightarrow \min, \int_0^1 tx dt = 0, x(0) = 0, x(1) = 1;$

в)  $\int_0^1 (\dot{x}^2 + x^2) dt \rightarrow \min, \int_0^1 xe^{-t} dt = (1 - 3e^{-2})/4, x(0) = 0, x(1) = 1/e;$

г)  $\int_0^\pi x \sin(t) dt \rightarrow \min, \int_0^\pi \dot{x}^2 dt = 3\pi/2, x(0) = 0, x(\pi) = \pi;$

д)  $\int_0^\pi \dot{x}^2 dt \rightarrow \min, \int_0^\pi x \sin(t) dt = 2 + \pi, \int_0^\pi x \cos(t) dt = \pi/2, x(0) = 2, x(\pi) = 0;$

е)  $\int_0^1 \dot{x}^2 dt \rightarrow \min, \int_0^1 x dt = 1, x(0) = 0, x(1) = 0.$