

ТИПОВОЙ РАСЧЕТ
«Дифференциальное исчисление
функции одной действительной переменной»

Задание 1. Найти производную функции.

$$1.1. y = (2x + 2)^7 + \frac{1}{\sqrt[7]{x^2}} + \sqrt{8x}.$$

$$1.2. y = (2x + 5)^5 + \frac{1}{\sqrt[4]{x^3}} + \sqrt{3x}.$$

$$1.3. y = (7x - 5)^4 + \frac{1}{7\sqrt[5]{x^2}} + \sqrt{5x}.$$

$$1.4. y = (2x - 7)^6 + \frac{1}{\sqrt[6]{x^5}} + \sqrt{7x}.$$

$$1.5. y = (3x - 1)^7 + \frac{1}{\sqrt{x^7}} + \sqrt{2x}.$$

$$1.6. y = (2x - 3)^{10} + \frac{1}{\sqrt[7]{x^3}} + \sqrt{7x + 1}.$$

$$1.7. y = (9x + 5)^8 + \frac{1}{\sqrt[5]{x^3}} + \sqrt{15x}.$$

$$1.8. y = (3x + 5)^7 + \frac{1}{\sqrt[5]{x^7}} + \sqrt{13x - 1}.$$

$$1.9. y = (7x + 5)^{11} + \frac{1}{\sqrt[8]{x^3}} + \sqrt{17x}.$$

$$1.10. y = (5x - 1)^7 + \frac{1}{\sqrt{x^3}} + \sqrt{10x - 2}.$$

$$1.11. y = (3x + 2)^5 + \frac{1}{\sqrt[7]{x^4}} + \sqrt{11x}.$$

$$1.12. y = (3x - 5)^6 + \frac{1}{\sqrt[7]{x^5}} + \sqrt{13x - 3}.$$

$$1.13. y = (7x + 2)^9 + \frac{1}{\sqrt[11]{x^7}} + \sqrt{2x + 5}.$$

$$1.14. y = (9x - 5)^{10} + \frac{1}{\sqrt[13]{x^8}} + \sqrt{3x + 8}.$$

$$1.15. y = (5 + 11x)^7 + \frac{4}{\sqrt[7]{x^3}} + \sqrt{5x - 2}.$$

$$1.16. y = (3 - 4x)^8 + \frac{3}{\sqrt[7]{x^6}} + \sqrt{13x}.$$

$$1.17. y = (5 - 2x)^{13} + \frac{11}{x^3\sqrt{x^2}} + \sqrt{2x + 5}.$$

$$1.18. y = (1 - 7x)^7 + \frac{5}{\sqrt[7]{x^9}} + \sqrt{2x - 15}.$$

$$1.19. y = (1 - 5x)^5 + \frac{1}{8\sqrt[10]{x^7}} + \sqrt{8x + 2}.$$

$$1.20. y = (9 - 2x)^6 + \frac{1}{\sqrt[9]{x^8}} + \sqrt{5x + 17}.$$

$$1.21. y = (3 - 5x)^7 + \frac{1}{2x^5\sqrt{x^2}} + \sqrt{15x}.$$

$$1.22. y = (7 - 2x)^6 + \frac{10}{\sqrt[9]{x^7}} + \sqrt{3x - 3}.$$

$$1.23. y = (3x - 2)^5 + \frac{1}{5\sqrt[8]{x^5}} + \sqrt{3 - 5x}.$$

$$1.24. y = (2x - 10)^8 + \frac{1}{\sqrt[11]{x^3}} + \sqrt{3x + 2}.$$

$$1.25. y = (2x - 3)^7 + \frac{3}{x^7\sqrt{x^2}} + \sqrt{2 - 11x}.$$

$$1.26. y = (5x + 5)^7 + \frac{1}{\sqrt[12]{x^7}} + \sqrt{15x - 3}.$$

$$1.27. y = (5x + 1)^5 + \frac{5}{\sqrt{x^7}} + \sqrt{11x - 7}.$$

$$1.28. y = (3x + 3)^3 + \frac{1}{\sqrt[3]{x^7}} + \sqrt{9x}.$$

$$1.29. y = (7x + 2)^4 + \frac{1}{7x^7\sqrt{x^3}} + \sqrt{10x}.$$

$$1.30. y = (5x + 7)^5 + \frac{7}{x^2\sqrt{x^3}} + \sqrt{11x+1}.$$

Задание 2. Найти производную функции.

$$2.1. y = \ln(\sqrt{1+x} + x^2) + \ln 4.$$

$$2.2. y = \ln(\sqrt{1+2x} + 5x^2) + \ln 5.$$

$$2.3. y = \ln(\sqrt{2x-5} - x^3) + \ln 10.$$

$$2.4. y = \ln(\sqrt{1+5x} - 3x) + \ln 7.$$

$$2.5. y = \ln(\sqrt{x-7} + 3x^3) + \ln 3.$$

$$2.6. y = 3\ln(\sqrt{7x-3} + x^3) + \ln 8.$$

$$2.7. y = \frac{1}{3}\ln(\sqrt{2x-3} + 7x^2) + \ln 9.$$

$$2.8. y = 5\ln(\sqrt{2x+2} + 3x^3) + \ln 11.$$

$$2.9. y = \frac{1}{5}\ln(\sqrt{3x-1} + 5x) + \ln 13.$$

$$2.10. y = 2\ln(\sqrt{3x+1} + 6x^2) + \ln 15.$$

$$2.11. y = \frac{1}{2}\ln(\sqrt{2-3x} - 5x^3) + \ln 12.$$

$$2.12. y = \ln(\sqrt{5+3x} - 10x^2) + \ln 2.$$

$$2.13. y = \ln(\sqrt{3+5x} + x^5) + \ln 6.$$

$$2.14. y = \ln(\sqrt{7-x} - x^2) + \ln 14.$$

$$2.15. y = \ln(\sqrt{5-2x} - 3x^2) + \ln 15.$$

$$2.16. y = \ln(\sqrt{2+2x} + 3x^2) + \ln 16.$$

$$2.17. y = \ln(\sqrt{2x-5} - 7x^2) + \ln 3.$$

$$2.18. y = \ln(\sqrt{2x+7} + 7x^3) + \ln 19.$$

$$2.19. y = 7\ln(\sqrt{7x-1} + 7x^2) + \ln 20.$$

$$2.20. y = \frac{1}{7}\ln(\sqrt{1-7x} - x^5) + \ln 15.$$

$$2.21. y = \ln(x^3 - \sqrt{3-3x} + 3x^3) + \ln 3.$$

$$2.22. y = \ln(x^2 + \sqrt{3x+11}) + \ln 5.$$

$$2.23. y = \ln(\sqrt{1-x-x^2}) + \ln 11.$$

$$2.24. y = 9 \ln(\sqrt{2+x} + 5x^3) + \ln 10.$$

$$2.25. y = \frac{1}{9} \ln(\sqrt{3-x-x^5}) + \ln 13.$$

$$2.26. y = 3 \ln(\sqrt{5+2x-x^3}) + \ln 7.$$

$$2.27. y = \frac{1}{3} \ln(\sqrt{1+9x-5x^2}) + \ln 9.$$

$$2.28. y = \ln(\sqrt{9x-3} + 9x^3) + \ln 5.$$

$$2.29. y = \ln(\sqrt{5+7x+11x^5}) + \ln 7.$$

$$2.30. y = \ln(\sqrt{5-x} + 17x^2) + \ln 2.$$

Задание 3. Найти производную функции.

$$3.1. y = \cos 5x \cdot 3^{9x} (x^2 + 4).$$

$$3.2. y = \sin(3x+1) \cdot 5^{3x} (x^2 - 3).$$

$$3.3. y = \sin 2x \cdot 2^{7x} (x^2 - 1).$$

$$3.4. y = \cos(3x-5) \cdot 3^{3x} (x^2 - 5).$$

$$3.5. y = \cos 7x \cdot 5^{9x} (4 - x^2).$$

$$3.6. y = \sin(5x+1) \cdot 2^{5x} (1 + x^2).$$

$$3.7. y = \cos(3x+2) \cdot 2^{9x} (1 - 7x^2).$$

$$3.8. y = \cos 3x \cdot 5^{7x} (3 - 5x^2).$$

$$3.9. y = \sin(2x-7) \cdot 9^{3x} (x^2 + 11).$$

$$3.10. y = \sin 5x \cdot 7^{3x} (x^2 + 5).$$

$$3.11. y = \sin(2-3x) \cdot 3^{5x} (2x^2 + 3).$$

$$3.12. y = \cos(1-3x) \cdot 7^{5x} (3x^2 - 2).$$

$$3.13. y = \cos(3x+7) \cdot 4^{7x} (5 - x^2).$$

$$3.14. y = \cos 11x \cdot 3^{7x} (11 + 3x^2).$$

$$3.15. y = \sin(7x-3) \cdot 2^{7x} (x^2 + 9).$$

$$3.16. y = \sin(3-5x) \cdot 2^{9x} (9 - x^2).$$

$$3.17. y = \sin 11x \cdot 2^{3x}(x^2 - 13).$$

$$3.18. y = \operatorname{tg} 5x \cdot 7^{9x}(3x^2 - 4).$$

$$3.19. y = \operatorname{tg} 7x \cdot 5^{9x}(4 - 7x^2).$$

$$3.20. y = \operatorname{tg} 2x \cdot 9^{5x}(4 - x^2).$$

$$3.21. y = \operatorname{ctg} 7x \cdot 5^{7x}(1 - 5x^2).$$

$$3.22. y = \operatorname{ctg}(3x + 2) \cdot 5^{2x}(2 + x^2).$$

$$3.23. y = \operatorname{ctg}(5x - 1) \cdot 11^x(3 - 2x^2).$$

$$3.24. y = \operatorname{tg} 9x \cdot 4^{2x}(x^2 + 15).$$

$$3.25. y = \operatorname{tg}(2x - 1) \cdot 4^{3x}(7 - 2x^2).$$

$$3.26. y = \operatorname{tg} 11x \cdot 7^{2x}(x^2 - 11).$$

$$3.27. y = \operatorname{ctg} 2x \cdot 9^{3x}(2x^2 - 7).$$

$$3.28. y = \operatorname{ctg}(2 - 3x) \cdot 4^{7x}(15 - x^2).$$

$$3.29. y = \operatorname{ctg}(2 - 3x) \cdot 4^{7x}(15 - x^2).$$

$$3.30. y = \operatorname{tg}(5x + 1) \cdot 5^{2x}(3x^2 - 3).$$

Задание 4. Найти производную функции.

$$4.1. y = \frac{x^3 + \sin x}{x^2 + 5} + e^{-2}.$$

$$4.2. y = \frac{x - \cos x}{5 - x^2} + e^{-5}.$$

$$4.3. y = \frac{2 \sin x - x^3}{x^2 + 3} + e^{-3}.$$

$$4.4. y = \frac{x^3 - \cos x}{x^2 - 3} + e^{-4}.$$

$$4.5. y = \frac{\operatorname{tg} x - x^5}{x^2 + 1} + e^{-7}.$$

$$4.6. y = \frac{x^3 - \operatorname{tg} x}{1 - x^2} + e^{-6}.$$

$$4.7. y = \frac{\operatorname{tg} x - 2x}{\operatorname{tg} x + 3} + e^{11}.$$

$$4.8. y = \frac{x^2 + 3 \sin x}{x^2 - 5} + e^{-11}.$$

$$4.9. y = \frac{x^5 - \cos 5x}{3 - x^2} + e^{-13}.$$

$$4.10. y = \frac{\sin 3x - x^3}{x^5 + 1} + e^9.$$

$$4.11. y = \frac{x^3 + \cos 3x}{x^3 - 1} + e^{-9}.$$

$$4.12. y = \frac{x^3 + \operatorname{tg} x}{x^2 + 1} + e^{13}.$$

$$4.13. y = \frac{\operatorname{tg} x - x^2}{x^5 - 3} + e^5.$$

$$4.14. y = \frac{x^2 - \operatorname{ctg} x}{x^2 + 2} + e^2.$$

$$4.15. y = \frac{x^4 + \operatorname{ctg} x}{x^2 - 7} + e^3.$$

$$4.16. y = \frac{7 - x^2}{\sin x + \cos x} + e^4.$$

$$4.17. y = \frac{x^3 - 5}{\cos x - \sin x} + e^7.$$

$$4.18. y = \frac{x^5 - 5}{x^3 - \cos x} + e^{-10}.$$

$$4.19. y = \frac{5 + x^4}{x^4 + \cos x} + e^{10}.$$

$$4.20. y = \frac{\operatorname{tg} x + x^4}{x^2 + 11} + e^{12}.$$

$$4.21. y = \frac{\sin x + \cos x}{\cos x - \sin x} + e^{-12}.$$

$$4.22. y = \frac{\operatorname{tg} x + 1}{\operatorname{tg} x - 1} + e^{15}.$$

$$4.23. y = \frac{x^5 + \sin 2x}{x^2 + 2} + e^{-15}.$$

$$4.24. y = \frac{\cos x - \sin x}{\cos x + \sin x} + 2e^3.$$

$$4.25. y = \frac{\operatorname{tg} 3x}{x^2 + 7} + 3e^{-5}.$$

$$4.26. y = \frac{\operatorname{tg} 2x + x}{x^3 - 5} + 2e^{-7}.$$

$$4.27. y = \frac{x^5 + 1}{\sin x + 3} + 7e^{-3}.$$

$$4.28. y = \frac{\operatorname{ctg} x - 5}{x^5 + 5} + 5e^{-4}.$$

$$4.29. y = \frac{2x^7 + 1}{3 \cos x - x} + 4e^{-9}.$$

$$4.30. y = \frac{3x^3 + \sin 2x}{x^5 - 3} + 3e^5.$$

Задание 5. Найти производную функции.

$$5.1. y = \cos^5(2^x + x) + \operatorname{tg} \frac{1 - e^x}{1 + e^x} + \cos \ln 2.$$

$$5.2. y = \sin^6(x - 3^x) + \operatorname{tg} \frac{e^{2x}}{1 - e^x} + \sin \ln 2.$$

$$5.3. y = \cos^3(x - 2^x) + \operatorname{ctg} \frac{e^x}{1 + e^x} + \cos \ln 3.$$

$$5.4. y = \sin^5(5^x + 1) + \operatorname{ctg} \frac{1 + e^{2x}}{e^{2x}} + \operatorname{ctg} \ln 3.$$

$$5.5. y = \cos^6(1 - 5^x) + \operatorname{tg} \frac{e^x}{1 - e^x} + \cos \ln 3.$$

$$5.6. y = \cos^4(x + 5^x) + \operatorname{ctg} \frac{1 - e^x}{e^x} + \sin \ln 3.$$

$$5.7. y = \sin^3(x - 5^x) + \operatorname{tg} \frac{e^{3x}}{1 - e^x} + \operatorname{tg} \ln 5.$$

$$5.8. y = \operatorname{tg}^4(x + 3^x) + \sin \frac{1 - e^x}{1 + e^x} + \operatorname{ctg} \ln 5.$$

$$5.9. y = \operatorname{tg}^6(x + 2^x) + \cos \frac{1 - e^x}{1 + e^x} + \cos \ln 5.$$

$$5.10. y = \cos^2(2^x - 3) + \sin \frac{e^{2x}}{1 + e^x} + \sin \ln 5.$$

$$5.11. y = \operatorname{tg}^2(3 + 2^x) - \cos \frac{1 + e^{2x}}{e^{2x}} + \operatorname{tg} \ln 9.$$

$$5.12. y = \operatorname{ctg}^2(3 - 5^x) - \sin \frac{e^{3x}}{1 + e^{3x}} + \operatorname{ctg} \ln 9.$$

$$5.13. y = \cos^3(5^x - 2) - \sin \frac{1 - e^x}{e^{5x}} + \sin \ln 4.$$

$$5.14. y = \sin^2(2 - 3^x) + \cos \frac{e^{4x}}{1 + e^{4x}} + \cos \ln 4.$$

$$5.15. y = \operatorname{tg}^5(5^x - 3x) + \sin \frac{1 + e^{3x}}{e^{3x}} + \operatorname{tg} \ln 4.$$

$$5.16. y = \operatorname{ctg}^4(3x - 3^x) + \cos \frac{e^{3x}}{1 - e^{3x}} + \operatorname{ctg} \ln 4.$$

$$5.17. y = \sin^4(5^x - x) + \operatorname{tg} \frac{e^{2x}}{1 - e^{2x}} + \sin \ln 7.$$

$$5.18. y = \sin^7(7^x + x) - \operatorname{ctg} \frac{1 - e^{2x}}{1 + e^{2x}} + \operatorname{tg} \ln 7.$$

$$5.19. y = \cos^7(3^x - x) + \operatorname{tg} \frac{e^x + 1}{1 - e^x} + \operatorname{ctg} \ln 7.$$

$$5.20. y = \operatorname{tg}^7(7^x - x) + \sin \frac{1 - e^{3x}}{1 + e^{3x}} + \sin \ln 8.$$

$$5.21. y = \operatorname{ctg}^7(x - 7^x) + \cos \frac{1 + e^{5x}}{e^{5x}} + \cos \ln 8.$$

$$5.22. y = \cos^6(x - 5^x) + \operatorname{tg} \frac{e^{4x}}{1 - e^{4x}} + \operatorname{tg} \ln 8.$$

$$5.23. y = \sin^8(2x - 3^x) + \operatorname{tg} \frac{e^{4x} - 1}{e^{4x} + 1} + \operatorname{ctg} \ln 8.$$

$$5.24. y = \operatorname{tg}^8(3x + 2^x) + \sin \frac{1 + e^{4x}}{e^{4x}} + \sin \ln 9.$$

$$5.25. y = \operatorname{ctg}^8(2^x - 3x) + \cos \frac{e^{4x}}{1 + e^{4x}} + \cos \ln 9.$$

$$5.26. y = \cos^8(7^x + x) + \operatorname{tg} \frac{e^{5x} - 1}{e^{5x} + 1} + \operatorname{tg} \ln 9.$$

$$5.27. y = \sin^9(3x - 2^x) - \operatorname{ctg} \frac{e^{2x}}{1 - e^{2x}} + \operatorname{ctg} \ln 9.$$

$$5.28. y = \cos^9(5 - 3^x) + \operatorname{tg} \frac{e^{5x}}{1 - e^{5x}} + \sin \ln 10.$$

$$5.29. y = \operatorname{tg}^9(7 - 7^x) + \sin \frac{1 + e^{5x}}{1 - e^{5x}} + \cos \ln 10.$$

$$5.30. y = \operatorname{ctg}^9(3 + 3^x) + \cos \frac{1 - e^{5x}}{e^{5x}} + \operatorname{tg} \ln 10.$$

Задание 6. Найти производную функции.

$$6.1. y = \arccos \sqrt{1 - x^3}.$$

$$6.16. y = \frac{1}{3} \operatorname{tg}^3 x.$$

$$6.2. y = \operatorname{ctg} \sqrt{x}.$$

$$6.17. y = x^2 10^{2x}.$$

$$6.3. y = 2 \cos^2(x + 2).$$

$$6.18. y = \operatorname{arccctg} \frac{1 + x}{1 - x}.$$

$$6.4. y = \sqrt{1 - (\arccos x)^2}.$$

$$6.19. y = \ln^2 x - \ln(\ln x).$$

$$6.5. y = \frac{\sin x}{2 \cos^2 x}.$$

$$6.20. y = \sqrt{\ln x + 1} + \ln(\sqrt{x} + 1).$$

$$6.6. y = \frac{1}{(1 + \sin 4x)^3}.$$

$$6.21. y = \operatorname{tg}^2 5x.$$

$$6.7. y = \frac{1}{(1 + \cos 2x)^3}.$$

$$6.22. y = \sin^2(x^3).$$

$$6.8. y = \ln(x + \sqrt{x^2 + 4}).$$

$$6.9. y = \sin^4 5x.$$

$$6.10. y = \operatorname{arctg} x^2.$$

$$6.11. y = \ln \frac{1+x}{\sqrt{1+x^2}}.$$

$$6.12. y = \ln^2(1 - \cos x).$$

$$6.13. y = \arcsin \frac{1}{\sqrt{x}}.$$

$$6.14. y = \operatorname{arctg} 3^{\sqrt{x}}.$$

$$6.15. y = \ln(\arcsin \sqrt{x}).$$

$$6.23. y = \arcsin(\ln x).$$

$$6.24. y = \operatorname{arctg}\left(\ln \frac{1}{x}\right).$$

$$6.25. y = \ln \cos \frac{x-1}{x}.$$

$$6.26. y = \frac{1}{\ln^2 x}.$$

$$6.27. y = x \sin\left(\ln x - \frac{\pi}{4}\right).$$

$$6.28. y = \sqrt{\sin x^2}.$$

$$6.29. y = x^2 \sin \frac{1}{x}.$$

$$6.30. y = \frac{x}{1 + e^{\frac{1}{x}}}.$$

Задание 7. Найти производную функции.

$$7.1. y = x^{\operatorname{tg} x}.$$

$$7.2. y = \sqrt[x]{x}.$$

$$7.3. y = \left(\frac{x}{2}\right)^{2x}.$$

$$7.4. y = x^{e^x}.$$

$$7.5. y = x^{-x}.$$

$$7.6. y = (\sin x)^x.$$

$$7.7. y = x^{x^3}.$$

$$7.8. y = (\ln x)^x.$$

$$7.9. y = x^{\frac{1}{x}}.$$

$$7.10. y = (\cos x)^{\sin x}.$$

$$7.11. y = x^{\sin x}.$$

$$7.16. y = x^{\operatorname{ctg} x}.$$

$$7.17. y = \sqrt[2x]{x}.$$

$$7.18. y = x^{2^x}.$$

$$7.19. y = x^{x^3}.$$

$$7.20. y = x^{\operatorname{arctg} x}.$$

$$7.21. y = (\ln x)^{3^x}.$$

$$7.22. y = x^{\operatorname{arctg} x}.$$

$$7.23. y = x^{e^{\cos x}}.$$

$$7.24. y = x^{\sin x^3}.$$

$$7.25. y = x^{2x^2+1}.$$

$$7.26. y = x^{\operatorname{ctg} x}.$$

$$7.12. y = (\cos x)^{\frac{1}{x}}.$$

$$7.13. y = (x+1)^{\frac{1}{\sin x}}.$$

$$7.14. y = x^{\arcsin x}.$$

$$7.15. y = x^{\arccos x}.$$

$$7.27. y = (x^4 + 1)^{\operatorname{tg} x}.$$

$$7.28. y = \left(\frac{1}{x}\right)^{3x}.$$

$$7.29. y = (\operatorname{tg} x)^{4e^x}.$$

$$7.30. y = x^{\cos x^2}.$$

Задание 8. Найти производную функции.

$$8.1. y = \log_2 \frac{\sqrt{x} + 5}{x^2 + x + 1}.$$

$$8.2. y = \ln^2 \cos x.$$

$$8.3. y = \sqrt{\ln(3x^2 + 2x)}.$$

$$8.4. y = \arcsin \frac{x}{\sqrt{1+x^2}}.$$

$$8.5. y = \ln \frac{x}{\sqrt{1-x^2}}.$$

$$8.6. y = \ln^4 \frac{1}{x^2}.$$

$$8.7. y = \ln \operatorname{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right).$$

$$8.8. y = \log_3 (x^2 - 1).$$

$$8.9. y = \ln^2 \sin x.$$

$$8.10. y = \ln^2 (1 + \cos x).$$

$$8.11. y = \ln \frac{x^2}{1-x^2}.$$

$$8.12. y = \log_2 (3x^2 + 1).$$

$$8.13. y = \ln \operatorname{tg} x.$$

$$8.16. y = \ln(\sqrt{x-1} + \sqrt{x^2+1}).$$

$$8.17. y = \ln \left(\arccos \frac{1}{\sqrt{x}} \right).$$

$$8.18. y = \ln(\arccos \sqrt{1-e^{4x}}).$$

$$8.19. y = \log_2 \frac{1}{\sqrt{1-x^4}}.$$

$$8.20. y = \ln \arccos 2x.$$

$$8.21. y = \ln \ln^3 \ln^2 x.$$

$$8.22. y = \ln \frac{\ln x}{\sin \frac{1}{x}}.$$

$$8.23. y = \log_4 \log_2 \operatorname{tg} x.$$

$$8.24. y = \ln \frac{1+x}{\sqrt{1+x^2}}.$$

$$8.25. y = \ln^2 \arcsin \sqrt[3]{x}.$$

$$8.26. y = \ln^2 \cos^3 (4x-1).$$

$$8.27. y = \ln \ln \ln x.$$

$$8.28. y = \ln \sqrt{\frac{1+\operatorname{tg} x}{1-\operatorname{tg} x}}.$$

$$8.14. y = \left(1 + \ln \frac{1}{x}\right)^5.$$

$$8.29. y = \ln(x + \sqrt{x^2 - 1}).$$

$$8.15. y = \ln \ln \frac{1}{\sqrt{x}}.$$

$$8.30. y = \ln \frac{1 + x^2}{1 - x^2}.$$

Задание 9. Найти производную y'_x .

$$9.1. \begin{cases} x = e^t \cos t, \\ y = e^{2t} \sin t. \end{cases}$$

$$9.16. \begin{cases} x = a \cos^2 t, \\ y = b \sin^2 t. \end{cases}$$

$$9.2. \begin{cases} x = \frac{3t}{1+t^3}, \\ y = \frac{3t^2}{1+t^3}. \end{cases}$$

$$9.17. \begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t). \end{cases}$$

$$9.3. \begin{cases} x = \cos^3 t, \\ y = \sin^3 t. \end{cases}$$

$$9.18. \begin{cases} x = t \ln t, \\ y = \frac{\ln t}{t}. \end{cases}$$

$$9.4. \begin{cases} x = \ln \operatorname{ctg} t, \\ y = \frac{1}{\cos^2 t}. \end{cases}$$

$$9.19. \begin{cases} x = \sqrt{1-t^2}, \\ y = \arcsin t. \end{cases}$$

$$9.5. \begin{cases} x = e^{3t} \cos 2t, \\ y = e^t \sin 2t. \end{cases}$$

$$9.20. \begin{cases} x = \ln \sin t, \\ y = \sqrt{1-t^2}. \end{cases}$$

$$9.6. \begin{cases} x = \sqrt{1-t^2}, \\ y = \operatorname{tg} \sqrt{1+t}. \end{cases}$$

$$9.21. \begin{cases} x = \ln(t^2 + 1), \\ y = \sqrt{t^2 + 1}. \end{cases}$$

$$9.7. \begin{cases} x = \sqrt{1-t}, \\ y = \operatorname{tg} t. \end{cases}$$

$$9.22. \begin{cases} x = \operatorname{tg} t + \operatorname{ctg} t, \\ y = t \cos t. \end{cases}$$

$$9.8. \begin{cases} x = \ln \operatorname{tg} t, \\ y = \frac{1}{\sin^2 t}. \end{cases}$$

$$9.23. \begin{cases} x = \frac{t+1}{t}, \\ y = \frac{t-1}{t}. \end{cases}$$

$$9.9. \begin{cases} x = t - \sin t, \\ y = 1 - \cos t. \end{cases}$$

$$9.24. \begin{cases} x = 2(1 - \sin t), \\ y = 2(t - \cos t). \end{cases}$$

$$9.10. \begin{cases} x = (\arcsin t)^2, \\ y = \frac{t}{\sqrt{1-t^2}}. \end{cases}$$

$$9.25. \begin{cases} x = 2t \operatorname{tg} t, \\ y = 2 \sin^2 t + \sin 2t. \end{cases}$$

$$9.11. \begin{cases} x = \frac{1}{t+1}, \\ y = \left(\frac{t}{t+1}\right)^2. \end{cases}$$

$$9.26. \begin{cases} x = \cos^2 t, \\ y = t \operatorname{g}^2 t. \end{cases}$$

$$9.12. \begin{cases} x = \ln \sqrt{\frac{1-t}{1+t}}, \\ y = \sqrt{1-t^2}. \end{cases}$$

$$9.27. \begin{cases} x = \ln t, \\ y = \operatorname{arctg} t. \end{cases}$$

$$9.13. \begin{cases} x = \sin t, \\ y = \ln \cos t. \end{cases}$$

$$9.28. \begin{cases} x = \cos t, \\ y = \ln \sin t. \end{cases}$$

$$9.14. \begin{cases} x = \ln(t-2), \\ y = t + \sin t. \end{cases}$$

$$9.29. \begin{cases} x = \operatorname{arctg} t, \\ y = \frac{t^2}{2}. \end{cases}$$

$$9.15. \begin{cases} x = \ln(4+t^2), \\ y = \sqrt{4+t^2}. \end{cases}$$

$$9.30. \begin{cases} x = \sin t - t \cos t, \\ y = \cos t + t \sin t. \end{cases}$$

Задание 10. Найти y' , y'' .

$$10.1. y = \operatorname{arctg} x.$$

$$10.16. y = \frac{1}{x^2 - 4}.$$

$$10.2. y = e^{-x} \sin x.$$

$$10.17. y = x^2 \cos \frac{1}{x}.$$

$$10.3. y = (x-2)e^{2x}.$$

$$10.18. y = \ln \sqrt[3]{1+x^2}.$$

$$10.4. y = x^2 \ln x.$$

$$10.19. y = \frac{1+x}{\sqrt{x}}.$$

$$10.5. y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{\frac{-x}{a}} \right).$$

$$10.20. y = x^3 \ln x.$$

$$10.6. y = \sqrt{2x-x^2}.$$

$$10.21. y = \ln \cos x.$$

$$10.7. y = \cos e^x + \sin e^x.$$

$$10.8. y = x^2 \sin \frac{1}{x}.$$

$$10.9. y = \frac{1+x}{1-x}.$$

$$10.10. y = \ln \sin x.$$

$$10.11. y = \frac{1-x}{1+x}.$$

$$10.12. y = \ln(1+x^2).$$

$$10.13. y = \frac{x+1}{x^2+1}.$$

$$10.14. y = \operatorname{arctg} \frac{1}{x}.$$

$$10.15. y = \frac{1}{2} x^2 e^x.$$

$$10.22. y = \arcsin \frac{1}{x}.$$

$$10.23. y = x^2 \ln x.$$

$$10.24. y = (1+x^2)e^x.$$

$$10.25. y = \operatorname{arctg} x - x.$$

$$10.26. y = \frac{x^3}{x^2+12}.$$

$$10.27. y = \sqrt[3]{x+2}.$$

$$10.28. y = \ln(1+x^3).$$

$$10.29. y = \sin^2 x.$$

$$10.30. y = e^{-x^2}.$$

Задание 11. Составить уравнение касательной и нормали к кривым в точке x_0 .

$$11.1. y = \sqrt{5+2x}, \quad x_0 = 2.$$

$$11.2. y = \frac{x^2}{x+1}, \quad x_0 = 1.$$

$$11.3. y = \frac{1}{1+x^2}, \quad x_0 = 2.$$

$$11.4. y = \frac{\sin x}{2-x}, \quad x_0 = 0.$$

$$11.5. y = \frac{2x}{1+x^2}, \quad x_0 = \sqrt{2}.$$

$$11.6. y = \frac{3x+1}{2x-1}, \quad x_0 = -1.$$

$$11.7. y = \sqrt{5-x^2}, \quad x_0 = 1.$$

$$11.8. y = \sqrt{4+2x-x^2}, \quad x_0 = 3.$$

$$11.9. y = \operatorname{arctg} 2x, \quad x_0 = 0.$$

$$11.10. y = \frac{x^2 + 1}{x - 3}, \quad x_0 = 4.$$

$$11.11. y = \cos 2x - 2 \sin x, \quad x_0 = \pi.$$

$$11.12. y = \cos\left(2x - \frac{\pi}{2}\right) + 2, \quad x_0 = \frac{\pi}{2}.$$

$$11.13. y = \sqrt[3]{x-1}, \quad x_0 = 2.$$

$$11.14. y = \ln x, \quad x_0 = 1.$$

$$11.15. y = \frac{1}{x} \operatorname{arctg} x, \quad x_0 = 1.$$

$$11.16. y = 3^x + 3^{-2x}, \quad x_0 = 1.$$

$$11.17. y = \operatorname{tg} 2x, \quad x_0 = 0.$$

$$11.18. y = \sin(x + \pi) + 1, \quad x_0 = \frac{\pi}{4}.$$

$$11.19. y = x^2 \ln x, \quad x_0 = 1.$$

$$11.20. y = \cos^2 x, \quad x_0 = \frac{\pi}{4}.$$

$$11.21. y = x^2 e^{-x}, \quad x_0 = 1.$$

$$11.22. y = \sqrt{x} + 1, \quad x_0 = 4.$$

$$11.23. y = \ln(2e - x), \quad x_0 = e.$$

$$11.24. y = \frac{1}{3x + 2}, \quad x_0 = 2.$$

$$11.25. y = \cos x - \frac{2}{\pi} x^2, \quad x_0 = \frac{\pi}{2}.$$

$$11.26. y = \sqrt[3]{x^2} - 20, \quad x_0 = -8.$$

$$11.27. y = \frac{\sqrt{x}}{x + 4}, \quad x_0 = 4.$$

$$11.28. y = 8\sqrt[4]{x} - 70, \quad x_0 = 16.$$

$$11.29. y = (x^2 - 5x + 7)e^x, \quad x_0 = 0.$$

$$11.30. y = 2x + \frac{1}{x}, \quad x_0 = 1.$$

Задание 12. Составить уравнения касательных к графику функции

$y = \frac{ax + b}{cx + d}$, параллельных или перпендикулярных к прямой $Ax + By + C = 0$.

12.1. $y = \frac{x+1}{x-1}$, параллельно прямой $2x + y - 2 = 0$.

12.2. $y = \frac{x-3}{x+1}$, параллельно прямой $x - y + 3 = 0$.

12.3. $y = \frac{x-3}{x+1}$, параллельно прямой $4x - y + 3 = 0$.

12.4. $y = \frac{-x+3}{x+1}$, параллельно прямой $x + y + 4 = 0$.

12.5. $y = \frac{-x+3}{x+1}$, параллельно прямой $4x + y - 3 = 0$.

12.6. $y = \frac{3x+2}{2x+2}$, параллельно прямой $2x - y + 4 = 0$.

12.7. $y = \frac{-3x+2}{-2x+4}$, параллельно прямой $2x + y - 4 = 0$.

12.8. $y = \frac{5x-1}{3x+3}$, параллельно прямой $2x - y + 7 = 0$.

12.9. $y = \frac{5x+3}{x+6}$, параллельно прямой $6x - 2y + 3 = 0$.

12.10. $y = \frac{5x+3}{-x-6}$, параллельно прямой $6x + 2y - 4 = 0$.

12.11. $y = \frac{4x+1}{4x+5}$, параллельно прямой $8x - 2y + 5 = 0$.

12.12. $y = \frac{4x-1}{4x-5}$, параллельно прямой $8x + 2y - 5 = 0$.

12.13. $y = \frac{6x-5}{3x+5}$, параллельно прямой $10x - 2y + 15 = 0$.

12.14. $y = \frac{7x+3}{2x-2}$, параллельно прямой $10x + 2y - 13 = 0$.

12.15. $y = \frac{9x-3}{x+1}$, параллельно прямой $3x - y + 7 = 0$.

12.16. $y = \frac{x-8}{x+4}$, перпендикулярно к прямой $6x + 2y - 7 = 0$.

12.17. $y = \frac{4x+6}{4x+3}$, перпендикулярно к прямой $6x - 2y + 7 = 0$.

12.18. $y = \frac{3x+1}{x+3}$, перпендикулярно к прямой $4x + 2y - 9 = 0$.

12.19. $y = \frac{-5x+1}{-2x+4}$, перпендикулярно к прямой $4x - 2y + 9 = 0$.

12.20. $y = \frac{7x+1}{2x-2}$, перпендикулярно к прямой $3x + 3y + 1 = 0$.

12.21. $y = \frac{8x+4}{6x+1}$, перпендикулярно к прямой $-4x + y - 7 = 0$.

12.22. $y = \frac{8x-4}{6x-1}$, перпендикулярно к прямой $8x + 2y - 11 = 0$.

12.23. $y = \frac{10x+7}{5x-9}$, перпендикулярно к прямой $10x - 2y + 21 = 0$.

12.24. $y = \frac{10x+7}{-5x+9}$, перпендикулярно к прямой $5x + y - 10,5 = 0$.

12.25. $y = -\frac{3x+3}{x+7}$, перпендикулярно к прямой $4x - 2y + 19 = 0$.

12.26. $y = \frac{3x-1}{x-2}$, перпендикулярно к прямой $10x - 2y + 9 = 0$.

12.27. $y = \frac{6x-1}{2x+5}$, перпендикулярно к прямой $4x + 2y - 15 = 0$.

12.28. $y = \frac{2x+2}{2x-1}$, перпендикулярно к прямой $12x - 2y + 21 = 0$.

12.29. $y = \frac{2x+2}{-2x+1}$, перпендикулярно к прямой $12x + 2y - 21 = 0$.

12.30. $y = \frac{3x-1}{x+2}$, перпендикулярно к прямой $14x + 2y - 25 = 0$.

Задание 13. Тело движется прямолинейно по закону $S(t)$. Найти скорость и ускорение тела в момент времени t_0 .

13.1. $S(t) = t^2 + 8\sqrt{(t+3)^3} - 2t + 1$, $t_0 = 1$.

- 13.2. $S(t) = 2t^2 + 4\sqrt{(t+2)^3} + 3t + 4,$ $t_0 = 2.$
- 13.3. $S(t) = 5t^2 - \sqrt{(t-3)^3} + t - 1,$ $t_0 = 4.$
- 13.4. $S(t) = 4t^2 + \sqrt{(t+3)^3} - 2t + 7,$ $t_0 = 0.$
- 13.5. $S(t) = t^3 + \sqrt[3]{(t-1)^4} - \frac{3}{2}t^2 + 2,$ $t_0 = 2.$
- 13.6. $S(t) = 3t^2 + 3\sqrt[3]{(t+2)^4} + 4t - 3,$ $t_0 = 6.$
- 13.7. $S(t) = 2t^3 - 2\sqrt{(t-3)^3} - \frac{t^2}{2} - 5,$ $t_0 = 4.$
- 13.8. $S(t) = \frac{t^3}{3} + 6\sqrt[3]{(t+7)^4} - t^2 - 3,$ $t_0 = 1.$
- 13.9. $S(t) = \frac{2}{3}t^3 + \sqrt{(t-1)^3} - 2t^2 + 2t - 3,$ $t_0 = 2.$
- 13.10. $S(t) = 7t^2 - 4\sqrt{(t+5)^3} - 3t + 10,$ $t_0 = 4.$
- 13.11. $S(t) = \frac{t^3}{6} - 3\sqrt{(t+3)^3} + 8t + 10,$ $t_0 = 4.$
- 13.12. $S(t) = 7t^2 - 9\sqrt[3]{(t-1)^4} - 5t - 3,$ $t_0 = 2.$
- 13.13. $S(t) = \frac{5}{6}t^3 + 4\sqrt{(t-3)^3} - 8t - 7,$ $t_0 = 4.$
- 13.14. $S(t) = \frac{3}{2}t^2 + 5\sqrt{(t+1)^3} - 9t - 5,$ $t_0 = 3.$
- 13.15. $S(t) = 4t^2 - 9\sqrt[3]{(t+3)^4} + 9t + 1,$ $t_0 = 5.$
- 13.16. $S(t) = \frac{2}{3}t^2 - 3\sqrt[3]{(t-4)^4} - 2t^2 - t - 4,$ $t_0 = 5.$
- 13.17. $S(t) = 5t^2 - 3\sqrt{(t+4)^3} - 4t - 1,$ $t_0 = 5.$
- 13.18. $S(t) = 4t^2 + 5\sqrt{(t-6)^3} - 80t + 3,$ $t_0 = 7.$
- 13.19. $S(t) = 10t^2 - 3\sqrt[3]{(t+1)^4} + 7t + 7,$ $t_0 = 0.$
- 13.20. $S(t) = 2t^2 - 10\sqrt{(t-2)^3} + 90t - 4,$ $t_0 = 6.$
- 13.21. $S(t) = 5t^2 + 3\sqrt[3]{(t-1)^4} - 20t + 1,$ $t_0 = 9.$

$$13.22. S(t) = t^3 - 6\sqrt[3]{(t+6)^4} + 93t - 4, \quad t_0 = 2.$$

$$13.23. S(t) = t^2 + 8\sqrt{(t+3)^3} - 2t + 1, \quad t_0 = 7.$$

$$13.24. S(t) = \frac{7}{18}t^3 - 3\sqrt{(t-2)^4} - 2t^2 + 10, \quad t_0 = 3.$$

$$13.25. S(t) = \frac{5}{9}t^3 - 2\sqrt[3]{(t+25)^4} + 12t^2 + 200, \quad t_0 = 1.$$

$$13.26. S(t) = 6t^2 - 3\sqrt{(t+2)^3} + 4t - 3, \quad t_0 = 2.$$

$$13.27. S(t) = 3t^2 - 6\sqrt{(t+1)^3} + 7t + 9, \quad t_0 = 0.$$

$$13.28. S(t) = \frac{t^3}{3} + 3\sqrt[3]{(t+1)} - 2t^2 + t - 1, \quad t_0 = 0.$$

$$13.29. S(t) = \frac{t^2}{4} + \sqrt{(t-2)^3} + 2t - 7, \quad t_0 = 11.$$

$$13.30. S(t) = \frac{t^3}{6} + 3\sqrt[3]{(t+8)^4} + \frac{t^2}{2} - 38, \quad t_0 = 0.$$

Задание 14. Найти приращение Δy и дифференциал dy функции $y = f(x)$ при переходе от точки x_0 к x .

$$14.1. y(x) = 3x^2 - 4x + 5, \quad x_0 = 1, \quad x = 1,2.$$

$$14.2. y(x) = 4x^2 - 3x + 2, \quad x_0 = 1, \quad x = 1,3.$$

$$14.3. y(x) = 3x^2 - x, \quad x_0 = 1, \quad x = 1,2.$$

$$14.4. y(x) = 2x^2 + 5x - 1, \quad x_0 = 3, \quad x = 3,4.$$

$$14.5. y(x) = x^3 - 7x + 3, \quad x_0 = 0, \quad x = 0,3.$$

$$14.6. y(x) = (x+9)^3, \quad x_0 = 2, \quad x = 2,3.$$

$$14.7. y(x) = \sqrt{x}, \quad x_0 = 0,25, \quad x = 0,251.$$

$$14.8. y(x) = 2x^3 + x^2 + 3x, \quad x_0 = 1, \quad x = 1,2.$$

$$14.9. y(x) = \frac{x}{1+x^2}, \quad x_0 = 1, \quad x = 1,3.$$

$$14.10. y(x) = x^4 - x + 8, \quad x_0 = 0, \quad x = 0,4.$$

- 14.11. $y(x) = (x+3)(x^2 - 2)$, $x_0 = 3$, $x = 3,5$.
- 14.12. $y(x) = 10x + 0,3$, $x_0 = 1$, $x = 1,2$.
- 14.13. $y(x) = x^2 - 5x - 7$, $x_0 = -2$, $x = -2,2$.
- 14.14. $y(x) = \frac{1}{4}x^4 + x^2 - 1$, $x_0 = 1$, $x = 1,6$.
- 14.15. $y(x) = (x^2 - 3)^4$, $x_0 = 0$, $x = 0,2$.
- 14.16. $y(x) = 5x^2 + 7x$, $x_0 = 5$, $x = 5,9$.
- 14.17. $y(x) = x^3 - x^2 + 1$, $x_0 = 1$, $x = 1,2$.
- 14.18. $y(x) = x^3 + x^2 + x$, $x_0 = 3$, $x = 3,6$.
- 14.19. $y(x) = 9x^4 - 7x^2 + 9$, $x_0 = 1$, $x = 1,2$.
- 14.20. $y(x) = 3x^3 - 7x + 1$, $x_0 = 0$, $x = 0,2$.
- 14.21. $y(x) = 5x^3 - 2$, $x_0 = 5$, $x = 5,7$.
- 14.22. $y(x) = (x^2 + 2)^2$, $x_0 = 1$, $x = 1,2$.
- 14.23. $y(x) = x^2 - 13x$, $x_0 = 3$, $x = 3,5$.
- 14.24. $y(x) = \frac{x}{x^2 - 2}$, $x_0 = 1$, $x = 1,2$.
- 14.25. $y(x) = x^3 + 12x^2$, $x_0 = 0$, $x = 0,2$.
- 14.26. $y(x) = x^5$, $x_0 = 7$, $x = 7,12$.
- 14.27. $y(x) = x^5 + 2x - 8$, $x_0 = 2$, $x = 2,4$.
- 14.28. $y(x) = x^2 - 4x$, $x_0 = 1$, $x = 1,2$.
- 14.29. $y(x) = 9x^3 + \log_3 27$, $x_0 = 5$, $x = 5,6$.
- 14.30. $y(x) = 5x - 4x^3$, $x_0 = 1$, $x = 1,8$.

Задание 15. Вычислить приближенно (с помощью дифференциала).

15.1. $\sqrt[3]{8,06}$.

15.16. $\frac{1}{\sqrt{9,02}}$.

15.2. $\sqrt[4]{16,02}$.

15.3. $(1,021)^{11}$.

15.4. $\frac{1}{\sqrt{4,016}}$.

15.5. $\sqrt{0,98^3}$.

15.6. $\sqrt[3]{(1,03)^2}$.

15.7. $(2,997)^5$.

15.8. $\arcsin 0,08$.

15.9. $\sqrt[4]{17}$.

15.10. $\arctg 0,98$.

15.11. $\ln 1,01$.

15.12. $\sqrt[5]{31}$.

15.13. $\sqrt[3]{7,98}$.

15.14. $\sqrt[4]{81,02}$.

15.15. $(0,99)^{12}$.

15.17. $\sqrt{(1,02)^3}$.

15.18. $(3,02)^5$.

15.19. $\arcsin(-0,04)$.

15.20. $\sqrt[4]{81,01}$.

15.21. $\arctg 1,02$.

15.22. $\ln 0,99$.

15.23. $\sqrt[5]{242}$.

15.24. $\sqrt[3]{27,01}$.

15.25. $\sqrt[4]{15,99}$.

15.26. $(0,98)^{10}$.

15.27. $e^{0,2}$.

15.28. $\sqrt{1,01^3}$.

15.29. $\sqrt{120}$.

15.30. $e^{-0,3}$.

Задание 16. Записать многочлен Тейлора 3-й степени для данной функции $y = f(x)$ в окрестности точки x_0 .

16.1. $y = \sin^2 x, x_0 = 0$.

16.16. $y = \cos^2 x, x_0 = 0$.

16.2. $y = \ln x, x_0 = 1$.

16.17. $y = \ln 2x, x_0 = 1$.

16.3. $y = \log_5 x, x_0 = 5$.

16.18. $y = e^x, x_0 = 1$.

16.4. $y = e^{x^2}, x_0 = 0$.

16.19. $y = e^{-x}, x_0 = 0$.

16.5. $y = e^{x^2}, x_0 = 1$.

16.20. $y = x^4, x_0 = 1$.

$$16.6. y = \sin x, x_0 = \frac{\pi}{2}.$$

$$16.7. y = \sin 3x, x_0 = 0.$$

$$16.8. y = \sin 2x, x_0 = \frac{\pi}{2}.$$

$$16.9. y = e^{\sin x}, x_0 = 0.$$

$$16.10. y = e^{3x}, x_0 = 1.$$

$$16.11. y = x^5, x_0 = 1.$$

$$16.12. y = \sin 2x, x_0 = \frac{\pi}{2}.$$

$$16.13. y = \sin^2 x, x_0 = 1.$$

$$16.14. y = \log_5 x, x_0 = 1.$$

$$16.15. y = \cos 3x, x_0 = \frac{\pi}{2}.$$

$$16.21. y = \cos x, x_0 = \frac{\pi}{2}.$$

$$16.22. y = \cos 5x, x_0 = 0.$$

$$16.23. y = \cos 3x, x_0 = 0.$$

$$16.24. y = e^{\cos x}, x_0 = 0.$$

$$16.25. y = e^{x^2}, x_0 = 1.$$

$$16.26. y = \frac{1}{x}, x_0 = 1.$$

$$16.27. y = \cos^2 x, x_0 = 1.$$

$$16.28. y = e^{2x}, x_0 = 1.$$

$$16.29. y = e^x, x_0 = 1.$$

$$16.30. y = \ln x, x_0 = 2.$$

Задание 17. Вычислить предел, используя правило Лопиталю.

$$17.1. \lim_{x \rightarrow 0} \frac{\sin^2 3x}{x^2}.$$

$$17.2. \lim_{x \rightarrow 1} \frac{x-1}{\ln x}.$$

$$17.3. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}.$$

$$17.4. \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{x - \sin x}.$$

$$17.16. \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 2x}.$$

$$17.17. \lim_{x \rightarrow 0} \frac{1 - \cos ax}{1 - \cos bx}.$$

$$17.18. \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}.$$

$$17.19. \lim_{x \rightarrow \infty} \frac{\ln x}{x}.$$

$$17.5. \lim_{x \rightarrow 0} \frac{\ln x}{\operatorname{ctg} x}.$$

$$17.20. \lim_{x \rightarrow \pi} (\pi - x) \operatorname{tg} \frac{x}{2}.$$

$$17.6. \lim_{x \rightarrow 0+0} x^x.$$

$$17.21. \lim_{x \rightarrow 0} x \ln x.$$

$$17.7. \lim_{x \rightarrow 0} (\sin x)^{\operatorname{tg} x}.$$

$$17.22. \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\cos 3x}.$$

$$17.8. \lim_{x \rightarrow 0} \frac{e^{ax} - e^{bx}}{\sin x}.$$

$$17.23. \lim_{x \rightarrow \frac{\pi}{2a}} \frac{1 - \sin ax}{(2ax - \pi)^2}.$$

$$17.9. \lim_{x \rightarrow 0} \frac{x - \operatorname{arctg} x}{x^3}.$$

$$17.24. \lim_{x \rightarrow 0} \frac{a^x - e^x}{\operatorname{tg} x}.$$

$$17.10. \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\cos 3x}.$$

$$17.25. \lim_{x \rightarrow 1} \frac{\ln x}{1 - x^3}.$$

$$17.11. \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \operatorname{tg} x}{\cos 2x}.$$

$$17.26. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\ln(1 + 2x)}.$$

$$17.12. \lim_{x \rightarrow 1} \frac{\ln x}{1 - x^3}.$$

$$17.27. \lim_{x \rightarrow 1} x^{\frac{1}{1-x}}.$$

$$17.13. \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x.$$

$$17.28. \lim_{x \rightarrow 0} (\sin x)^{\operatorname{tg} x}.$$

$$17.14. \lim_{x \rightarrow 0} (e^{2x} + x)^{\frac{1}{x}}.$$

$$17.29. \lim_{x \rightarrow 0} \left(\frac{1}{x \sin x} - \frac{1}{x^2} \right).$$

$$17.15. \lim_{x \rightarrow 0} (1 - e^{2x}) \operatorname{ctg} x.$$

$$17.30. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\ln(1 + 2x)}.$$

Задание 18. Найти наибольшее и наименьшее значения функции $y = f(x)$ на отрезках.

18.1. $y(x) = \frac{1}{3}x^3 - 4x^2 + 12x + 1$, $[0;3]$, $[-1;1]$.

18.2. $y(x) = \frac{1}{4}x^4 + \frac{4}{3}x^3 - \frac{9}{2}x^2 - 36x$, $[-5;8]$, $[-1;4]$.

18.3. $y(x) = \frac{1}{3}x^3 - x^2 - 3x - 2$, $[-3;5]$, $[-4;0]$.

18.4. $y(x) = \frac{2}{4}x^4 + \frac{4}{3}x^3 - 16x^2 - 64x$, $[-5;5]$, $[-6;-1]$

18.5. $y(x) = 2x^3 - 8x^2 - 6x + 4$, $[-2;5]$, $[1;4]$.

18.6. $y(x) = \frac{49}{2}x^2 - \frac{8}{3}x^3 - 45x - 3$, $[0;6]$, $[-2;2]$.

18.7. $y(x) = \frac{1}{3}x^3 + \frac{7}{2}x^2 + 6x - 13$, $[0;-2]$, $[-7;0]$.

18.8. $y(x) = 7x^2 - \frac{1}{3}x^3 - 49x + 4$, $[-3;10]$, $[8;9]$.

18.9. $y(x) = \frac{1}{4}x^4 + \frac{13}{3}x^3 - \frac{1}{2}x^2 - 13x$, $[-15;2]$, $[-2;3]$.

18.10. $y(x) = \frac{31}{2}x^2 - x^3 + 22x - 7$, $[-1;12]$, $[2;13]$.

18.11. $y(x) = 2x^3 + 14x^2 + 32x - 3$, $[-5;0]$, $[-2.5;1]$.

18.12. $y(x) = \frac{25}{2}x^2 - \frac{1}{4}x^4 - \frac{4}{3}x^3 + 100x$, $[-7;7]$, $[3;6]$.

18.13. $y(x) = \frac{1}{3}x^3 + 5x^2 - 24x + 6$, $[-13;1]$, $[-15;7]$.

18.14. $y(x) = 7x^3 + \frac{5}{2}x^2 - 4x + 11$, $[-2;0]$, $\left[\frac{1}{5};3\right]$.

18.15. $y(x) = \frac{1}{3}x^3 + 3x^2 - 27x + 6$, $[-15;5]$, $[0;6]$.

18.16. $y(x) = 18x^2 - 396x - \frac{1}{4}x^4 + \frac{11}{3}x^3$, $[-8;12]$, $[0;7]$.

18.17. $y(x) = 12x^3 + 57x^2 + 18x - 27$, $[-4;5]$, $[-4;-2]$.

18.18. $y(x) = \frac{4}{3}x^3 + 10x^2 + 24x + 1$, $[-4;1]$, $[-2,5;0]$.

18.19. $y(x) = \frac{35}{3}x^3 + \frac{69}{2}x^2 - 36x + 17$, $[-3;2]$, $[0;7]$.

18.20. $y(x) = 50x^2 + 200x - \frac{1}{4}x^4 - \frac{2}{3}x^3$, $[-12;12]$, $[0;15]$.

18.21. $y(x) = 6x^3 - \frac{87}{2}x^2 + 84x - 11$, $[-2;5]$, $[2;7]$.

18.22. $y(x) = \frac{153}{2}x^2 - 5x^3 - 162x + 21$, $[-3;10]$, $[5;11]$.

18.23. $y(x) = \frac{2}{3}x^3 + \frac{35}{2}x^2 + 143x - 13$, $[-13;0]$, $[-9;-4]$.

18.24. $y(x) = x^3 + 28x^2 - 220x + 13$, $[-25;5]$, $[0;7]$.

18.25. $y(x) = \frac{1}{4}x^4 + \frac{7}{3}x^3 - \frac{121}{2}x^2 - 847x + 21$, $[-15;21]$, $[9;12]$.

18.26. $y(x) = \frac{26}{3}x^3 + 43x^2 - 72x - 1$, $[-5;2]$, $[-10;0]$.

18.27. $y(x) = \frac{85}{3}x^3 - 32x^2 - 77x + 2$, $[-2;2]$, $[1;9]$.

18.28. $y(x) = \frac{3}{2}x^4 + x^3 - 192x^2 - 192x + 72$, $[-10;10]$, $[2;9]$.

18.29. $y(x) = \frac{1}{3}x^3 + \frac{15}{2}x^2 - 496x - 7$, $[-32;17]$, $[0;3]$.

18.30. $y(x) = 494x - \frac{1}{3}x^3 - \frac{7}{2}x^2 + 5$, $[-30;20]$, $[15;25]$.

Задание 19. Найти асимптоты и схематически построить график функции.

$$19.1. y = \frac{x^3 - 3x^2 + 7}{x^4 - 4}.$$

$$19.2. y = \frac{x}{\sqrt{1 - x^2}}.$$

$$19.3. y = \sqrt{\frac{1-x}{x}}.$$

$$19.4. y = \frac{6(x^2 - 4)}{3x^2 + 8}.$$

$$19.5. y = \frac{\sqrt{1+x^2}}{x}.$$

$$19.6. y = \frac{x^2 - 2x}{x - 1}.$$

$$19.7. y = \frac{\sqrt{4x^4 + 1}}{|x|}.$$

$$19.8. y = x + \frac{1}{x^2}.$$

$$19.9. y = \frac{2x^4 + x^3 + 1}{x^3}.$$

$$19.10. y = \frac{x^2 - 6x + 3}{x - 3}.$$

$$19.11. y = \frac{x^5}{x^4 - 1}.$$

$$19.12. y = \frac{x^2 + 1}{\sqrt{x^2 - 1}}.$$

$$19.13. y = \frac{x}{x^2 - 4x + 3}.$$

$$19.14. y = \frac{x^2}{\sqrt{x^2 - 1}}.$$

$$19.16. y = \frac{4}{x^4 - 2x^2}.$$

$$19.17. y = \frac{(x+1)^3}{(x+2)^2}.$$

$$19.18. y = \sqrt{\frac{x}{x-2}}.$$

$$19.19. y = \frac{\sqrt{4+x^2}}{2x}.$$

$$19.20. y = \frac{\sqrt{9x^4 + 1}}{|x|}.$$

$$19.21. y = 2x - \frac{1}{x^2}.$$

$$19.22. y = \frac{4 + x^2}{x^3 - 9x}.$$

$$19.23. y = \frac{x}{2x - 1} + x.$$

$$19.24. y = \frac{2x^2 + 3x - 5}{x(x - 4)}.$$

$$19.25. y = \frac{x^2 + 5}{x^2 - 1} + 2x.$$

$$19.26. y = \frac{x^4}{(1+x)^2}.$$

$$19.27. y = \frac{1}{2x^2 + x - 1}.$$

$$19.28. y = \frac{x^2}{4 - x^2}.$$

$$19.29. y = \frac{2x^2 - 9}{\sqrt{x^2 - 1}}.$$

$$19.15. y = \frac{x^2 - 1}{x^2 - 5x + 6}.$$

$$19.30. y = \frac{x^2 - 11}{4x - 3}.$$

Задание 20. Провести полное исследование и построить график функции $y = f(x)$.

$$20.1. \text{ а) } y = \frac{x^2}{x - 2};$$

$$\text{б) } y = (x + 2)e^{-x};$$

$$\text{в) } y = \sqrt[3]{x+1} - \sqrt[3]{x-1}.$$

$$20.2. \text{ а) } y = \frac{x^2 - 4x + 1}{x - 4};$$

$$\text{б) } y = xe^x;$$

$$\text{в) } y = \sqrt[3]{x^2 - 2x}.$$

$$20.3. \text{ а) } y = \frac{x^2 - 5x}{1 - x};$$

$$\text{б) } y = (x - 2)e^{3-x};$$

$$\text{в) } y = \sqrt[3]{(x+1)^2} + \sqrt[3]{(x-1)^2}.$$

$$20.4. \text{ а) } y = \frac{6 - 2x^2}{x - 2};$$

$$\text{б) } y = (x - 1)e^{2x};$$

$$\text{в) } y = \frac{1}{\sqrt[3]{x+1}} + \frac{1}{\sqrt[3]{x-1}}.$$

$$20.5. \text{ а) } y = \frac{3x^2}{2 - x};$$

$$\text{б) } y = (2 - x)e^x;$$

$$\text{в) } y = \sqrt[3]{1 - x^3}.$$

$$20.6. \text{ а) } y = \frac{7x - x^2}{x - 3};$$

$$\text{б) } y = xe^{-2x};$$

$$\text{в) } y = \sqrt[3]{x+1} + \sqrt[3]{x-1}.$$

$$20.7. \text{ а) } y = \frac{x^2}{3 - x};$$

$$\text{б) } y = (x + 1)e^{2x};$$

$$\text{в) } y = \sqrt[3]{x^3 + 1} + \sqrt[3]{x^3 - 1}.$$

$$20.8. \text{ а) } y = \frac{x^2 + 16}{4x};$$

$$\text{б) } y = (3 - x)e^{x-2};$$

$$\text{в) } y = \frac{x^3}{\sqrt{x^4 + 1}}.$$

$$20.9. \text{ а) } y = \frac{x^2 - x + 1}{1 - x};$$

$$\text{б) } y = (4 - x)e^{x-3};$$

$$\text{в) } y = \frac{x}{\sqrt{x^2 + 1}}.$$

$$20.10. \text{ а) } y = \frac{x^2}{x + 2};$$

$$\text{б) } y = xe^{3x};$$

$$\text{в) } y = \frac{x^3}{\sqrt[3]{x^3 + 2}}.$$

$$20.11. \text{ а) } y = \frac{2x^2}{x - 1};$$

$$\text{б) } y = (x - 3)e^{-x};$$

$$\text{в) } y = \frac{x^3}{\sqrt[3]{x^3 - 4}}.$$

$$20.12. \text{ а) } y = \frac{3x^2}{x - 2};$$

$$\text{б) } y = (x - 5)e^{2x};$$

$$\text{в) } y = \frac{x^3}{\sqrt[3]{(x^3 + 2)^2}}.$$

$$20.13. \text{ а) } y = \frac{x^2 - 3x + 3}{x - 1};$$

$$\text{б) } y = xe^{-x};$$

$$\text{в) } y = \frac{x^2}{\sqrt{x^2 + 1}}.$$

$$\mathbf{20.14.} \text{ а) } y = \frac{x^2}{x+1};$$

$$\text{б) } y = (x+1)e^{-2x};$$

$$\text{в) } y = \frac{\sqrt[3]{x^3 + 2}}{x}.$$

$$\mathbf{20.15.} \text{ а) } y = \frac{3-x^2}{x+2};$$

$$\text{б) } y = (x+4)e^{2x};$$

$$\text{в) } y = \frac{x}{\sqrt{(x^3 + 1)^2}}.$$

$$\mathbf{20.16.} \text{ а) } y = \frac{x^2}{x-5};$$

$$\text{б) } y = (x+1)e^{-x};$$

$$\text{в) } y = \frac{x^2}{\sqrt{|x^2 - 1|}}.$$

$$\mathbf{20.17.} \text{ а) } y = \frac{x^2 - x + 1}{x-1};$$

$$\text{б) } y = xe^{2x-1};$$

$$\text{в) } y = \frac{\sqrt{|x^2 - 3|}}{x}.$$

$$\mathbf{20.18.} \text{ а) } y = \frac{-x^2 - 4}{2x};$$

$$\text{б) } y = (x-1)e^{-x};$$

$$\text{в) } y = \sqrt[3]{|x^2 - 1|}.$$

$$\mathbf{20.19.} \text{ а) } y = \frac{x^2}{4+x};$$

$$\text{б) } y = (x-3)e^{x-2};$$

$$\text{в) } y = \sqrt{|x^2 - 2|^3}.$$

$$20.20. \text{ а) } y = \frac{2x^2 - 6}{x - 2}; \quad \text{б) } y = (x - 4)e^{x-3};$$

$$\text{в) } y = \sqrt{8x^2 - x^4}.$$

$$20.21. \text{ а) } y = \frac{x^2}{5 - x}; \quad \text{б) } y = xe^{2-x};$$

$$\text{в) } y = \frac{x - 2}{\sqrt{x^2 + 1}}.$$

$$20.22. \text{ а) } y = \frac{x^2 - 4x + 1}{4 - x}; \quad \text{б) } y = (x + 2)e^{2x};$$

$$\text{в) } y = \sqrt[3]{x^3 - x^2 - x + 1}.$$

$$20.23. \text{ а) } y = \frac{x^2}{x - 3}; \quad \text{б) } y = (x - 4)e^x;$$

$$\text{в) } y = \sqrt{(x - 1)(x - 2)(x - 3)}.$$

$$20.24. \text{ а) } y = \frac{x^2 + 11}{x - 5}; \quad \text{б) } y = (x - 4)e^{-x};$$

$$\text{в) } y = \sqrt[3]{x^2} - \sqrt[3]{x^2 + 1}.$$

$$20.25. \text{ а) } y = \frac{x^2 + 4}{2x}; \quad \text{б) } y = (x - 1)e^x;$$

$$\text{в) } y = \sqrt{\frac{|1 + x|^3}{x}}.$$

$$20.26. \text{ а) } y = \frac{x^2 - 5x}{x - 1}; \quad \text{б) } y = (5 - x)e^{2x};$$

$$\text{в) } y = \frac{x}{\sqrt[3]{x^2 - 1}}.$$

$$20.27. \text{ а) } y = \frac{3x - x^2 - 3}{x - 1};$$

$$\text{б) } y = (x - 2)e^{2-x};$$

$$\text{в) } y = -\sqrt{8x^2 - x^4}.$$

$$20.28. \text{ а) } y = \frac{x^2}{2 - x};$$

$$\text{б) } y = (3 - x)e^{-x};$$

$$\text{в) } y = \frac{x^2 \sqrt{x^2 - 1}}{2x^2 - 1}.$$

$$20.29. \text{ а) } y = \frac{x^2 - 3}{x + 2};$$

$$\text{б) } y = (x - 4)e^{4-x};$$

$$\text{в) } y = 1 - x + \sqrt{\frac{x^3}{3 + x}}.$$

$$20.30. \text{ а) } y = \frac{2x^2}{1 - x};$$

$$\text{б) } y = (x - 1)e^{3x-1};$$

$$\text{в) } y = \sqrt[3]{\frac{x^2}{x+1}}.$$

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